



# INDIAN SCHOOL DARSAIT DEPARTMENT OF MATHEMATICS



**Subject : MATHEMATICS Topic : RELATIONS AND FUNCTIONS**

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**Name of the Student: \_\_\_\_\_**

**Class & Division:**

**Roll. Number:**

Sl.No.	Questions	Mark s
1.	Show that the relation R on the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b);  a-b  \text{ is even}\}$ , is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other. But, no element of $\{1,3,5\}$ is related to any element of $\{2,4\}$ . (RD Sharma page No; 1.17 )	6
2.	Show that the relation R on the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ , given by $R = \{(a,b) :  a-b  \text{ is multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1. (RD Sharma page No; 1.18)	4
3.	Prove that the relation R on the set $\mathbb{N} \times \mathbb{N}$ defined by $(a,b) R (c,d) \Leftrightarrow a+d = b+c$ for all $(a,b), (c,d) \in \mathbb{N} \times \mathbb{N}$ is an equivalence relation. (RD Sharma page No; 1.1)	6
4.	Let * be the binary operation on $\mathbb{Z}$ given by $a * b = a + b - 15$ . (i) Is * commutative? (ii) Is * associative? (iii) Does the identity element exist? If yes find it. (iv) Are the elements of $\mathbb{Z}$ invertible? If so, find the inverse. (Board Qn:)	4
5.	Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = x^2 + 1$ is not invertible. (Board Qn:)	1
6.	If $f(x) = x + 7, g(x) = x - 7, x \in \mathbb{R}, f \circ g$ (Board Qn:)	1
7.	If $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions defined as $f(x) =  x  + x$ and $g(x) =  x  - x, \forall x \in \mathbb{R}$ . Then find $f \circ g$ and $g \circ f$ . Hence find $f \circ g(-3), f \circ g(5)$ and $g \circ f(-2)$ . (Board paper 2016)	6
8.	Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$ . Show that $f: \mathbb{N} \rightarrow S$ , where S is the range of f, is invertible. Also find the inverse (Board paper 2015)	6
9.	Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$ . Show that $f: \mathbb{N} \rightarrow S$ , where S is the range of f, is invertible. Also find the inverse and $f^{-1}(31), f^{-1}(87)$ . (Board paper 2016)	6
10.	Let * be a binary operation on the set of all non-zero real numbers, given by $a * b = \frac{ab}{5}$ , for all $a, b \in \mathbb{R} - \{0\}$ . Find the value of x, given that $2 * (x * 5) = 10$ . (Board paper 2014)	1
11.	Let * be the binary operation on the set $\{0,1,2,3,4,5\}$ given by $a * b = a + b$ ; if $a+b < 6$ $a + b - 6$ ; if $a+b \geq 6$ Show that zero is the identity for this operation and each element a of the set is invertible with $6-a$ being the inverse of a. (Board Qn: 2011)	4
12.	If $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = x^2 + 2, g: \mathbb{R} \rightarrow \mathbb{R}$ $g(x) = \frac{x}{x-1}, x \neq -1$ . Then find $f \circ g$ and $g \circ f$ . Hence find $f \circ g(2), f \circ g(5)$ and $g \circ f(-3)$ . (Board paper 2014)	4

13.	Let $A = \mathbb{R} - \{3\}$ , $B = \mathbb{R} - \{1\}$ . Let $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$ , $\forall x \in A$ . Show that $f$ is bijective. Also find (1) $x$ , if $f^{-1}(x) = 4$ (2) $f^{-1}(7)$ . ( Board paper 2017)	6
14.	Let $A = \mathbb{R} \times \mathbb{R}$ and let $*$ be a binary operation on $A$ defined by $(a,b) * (c,d) = (ad + bc, bd)$ for all, $(a,b), (c,d) \in \mathbb{R} \times \mathbb{R}$ . 1) Show that $*$ is commutative on $A$ 2) Show that $*$ is associative on $A$ 3) Find the identity element of $*$ in $A$ ( Board paper 2017)	6
15.	Consider $f: \mathbb{R} \rightarrow (-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$ . Prove that $f(x)$ is invertible with  $f^{-1}(y) = \frac{\sqrt{54 + 5y} - 3}{5}$	6

Dear Children,  
There is no substitute for hard work.  
All the best. God Bless.