SET – 1

Subject Code: 041

COMMON PRE-BOARD EXAMINATION 2017-2018

MATHEMATICS

MARKING SCHEME

CLASS XII

Time Allowed: 3 hours Maximum Marks: 100

Sr.No	Answer	Marks
	SECTION.A	
1.	$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 4 & 1 \\ 0 & 1 & 1 \\ 4 & 7 & 1 \end{vmatrix} = \frac{1}{2}[2(1-7)-4(0-4)+1(0-4)] = \frac{1}{2}\times 0 = 0$ $\therefore$ Three points are collinear.	1
2.	$\frac{dx}{d\theta} = a(1+\cos\theta),\ \frac{dy}{d\theta} = a(\sin\theta) \Rightarrow \frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{\theta}{2}$	1
3.	$\frac{dy}{y^2} = -4x\,dx \Rightarrow \int \frac{dy}{y^2} = -4\int x\,dx \Rightarrow -\frac{1}{y} = -2x^2 + C$ When y=1, x = 0 then C = -1. $\Rightarrow y = \frac{1}{2x^2+1}$	1
4.	$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert} = \frac{3-4+10}{\sqrt{9+1+4}.\sqrt{1+16+25}} = \frac{9}{\sqrt{588}} \Rightarrow \theta = \cos^{-1}\left(\frac{9}{14\sqrt{3}}\right)$	1
	SECTION.B	
5	Put $x = \cos\theta \Rightarrow \sin^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{2}\right] = \sin^{-1}\left[\frac{\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}{2}\right]$ $= \sin^{-1}\left[\sin\frac{\theta}{2}\cos\frac{\pi}{4}+\cos\frac{\theta}{2}\sin\frac{\pi}{4}\right] \Rightarrow \sin^{-1}\left[\sin\left(\theta+\frac{\pi}{4}\right) \Rightarrow \theta+\frac{\pi}{4} \Rightarrow \frac{\pi}{4}+\cos^{-1}x\right]$	1 1
6	$\tan^{-1}x+\tan^{-1}y = \pi - \tan^{-1}z \Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \pi - \tan^{-1}z$ $\Rightarrow \frac{x+y}{1-xy} = -\tan\left(\tan^{-1}z\right) \Rightarrow \frac{x+y}{1-xy} = -z \Rightarrow x+y+y = xyz$	1 1
7	$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - \begin{bmatrix} 3x \\ 6y \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2-3x \\ y^2-6y \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix} \Rightarrow x^2-3x+2=0,\ y^2-6y+9=0$ $\Rightarrow (x-2)(x-1)=0,\ (y-3)^2=0 \Rightarrow x=1,2$ and $y=3$	½+ ½ 1

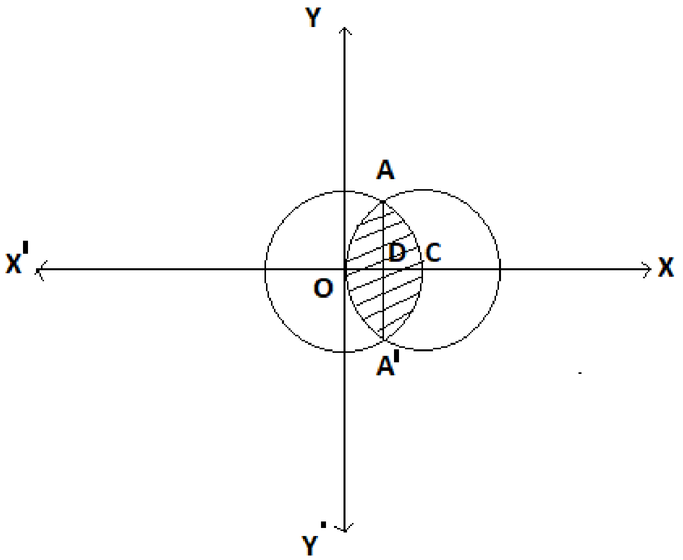
8	$y = 3x^4 - 4x \Rightarrow \frac{dy}{dx} = 12x^3 - 4 \Rightarrow \left.\frac{dy}{dx}\right\|_{x=4} = 764$ $\therefore$ Slope of tangent = 764, Slope of normal = -1/764	½ + ½ ½+ ½
9	$\text{I} = \int \frac{dx}{4-2x-x^2} = \int \frac{dx}{-(x^2+2x-4)} = \int \frac{dx}{-[(x+1)^2-5]}$ $\Rightarrow \int \frac{dx}{(\sqrt{5})^2-(x+1)^2} = \frac{1}{2\sqrt{5}} \log \left\|\frac{\sqrt{5}+(x+1)}{\sqrt{5}-(x+1)}\right\| + C$	½+ ½ 1
10	$\frac{dx}{dy} = x + y + 1 \Rightarrow \frac{dx}{dy} - x = y + 1 \Rightarrow I.F = e^{\int -dy} = e^{-y}$ $\Rightarrow xe^{-y} = \int (y+1)e^{-y} dy \Rightarrow xe^{-y} = -(y+1)e^{-y} - e^{-y} + C \Rightarrow x = -(y+2) + Ce^{y}$	1 1
11	$\vec{r} = 2\hat{a} - \hat{b} + 3\hat{c} \Rightarrow \vec{r} = 5\hat{i} + 15\hat{j} + 18\hat{k}$ $\|\vec{r}\| = \sqrt{5^2 + 15^2 + 18^2} = \sqrt{574}$units $\Rightarrow \hat{r} = \pm\frac{\vec{r}}{\|\vec{r}\|} = \pm\left[\frac{5\hat{i} + 15\hat{j} + 18\hat{k}}{\sqrt{574}}\right]$	½ ½ 1
12	n(S) = 36 , E = Doublets .n(E) =6, F = Total of two numbers is 10, n(F) = 3 $P(F) = 3/36$, $\text{n}(\text{E} \cap \text{F}) = 1 \Rightarrow \text{P}(\text{E} \cap \text{F}) = 1/36$ $\therefore P(\text{E}\|\text{F}) = \frac{\text{P}(\text{E} \cap \text{F})}{\text{P(F)}} = \frac{1}{3}$	 ½ ½ 1
	SECTION.C	
13	$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & cb & c^2+1 \end{vmatrix} \Rightarrow abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix}$ (Taking a, b, c from R_1, R_2 and R_3) $\begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$ (Multiplying C_1, C_2, C_3 by a, b and c) $\begin{vmatrix} 1+a^2+b^2+c^2 & ab & ac \\ 1+a^2+b^2+c^2 & b^2+1 & bc \\ 1+a^2+b^2+c^2 & cb & c^2+1 \end{vmatrix}$ $(C_1 \to C_1 + C_2 + C_3)$ $\Rightarrow (1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix} \Rightarrow (1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ $(R_2 \to R_2 - R_1, R_3 \to R_3 - R_1)$	½ ½ 1 1

	$\Rightarrow \left(1+a^2+b^2+c^2\right)\left[1\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}-0+0\right]=1+a^2+b^2+c^2$	1
14	$\lim_{x\to 1^-} f(x) = \lim_{x-1^-} (5ax-2b) = 5a-2b$ $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(3ax+b) = 3a+b$ F(x) is continous at x = 1, then we have $\lim_{x\to 1} f(x) = f(1)$ $\Rightarrow 5a-2b=11,\ 3a+b=11$. By solving we get a = 3 and b = 2	1 1 1+1
OR	$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{2} = \lim_{x\to 2}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{2}\times\frac{\sqrt{5x+2}+\sqrt{4x+4}}{\sqrt{5x+2}+\sqrt{4x+4}}$ $\Rightarrow \lim_{x\to 2}\frac{x-2}{(x-2)(\sqrt{5x+2}+\sqrt{4x+4})}$ $\Rightarrow \lim_{x\to 2}\frac{1}{\sqrt{5x+2}+\sqrt{4x+4}} = \frac{1}{\sqrt{12}+\sqrt{12}} = \frac{1}{4\sqrt{3}}$ Also f(2) = k. let f(x) be continous at x = 2. $\lim_{x\to 2} f(x) = f(2) \Rightarrow k = \frac{1}{4\sqrt{3}}$	1 1 1 1
15	Given that $\cos y = x\cos(a+y) \Rightarrow \frac{d}{dx}[\cos y] = \frac{d}{dx}[x\cos(a+y)]$ $\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y).\frac{d}{dx}(x) + x.\frac{d}{dx}[\cos(a+y)]$ $\Rightarrow --\sin y\frac{dy}{dx} = \cos(a+y) + x.[-\sin(a+y)]\frac{dy}{dx}$ $\Rightarrow [x\sin(a+y)-\sin y]\frac{dy}{dx} = \cos(a+y)$ ………..(1) $(1) \Rightarrow \left[\frac{\cos y}{\cos(a+y)}.\sin(a+y)-\sin y\right]\frac{dy}{dx} = \cos(a+y)$ $\Rightarrow [\cos y.\sin(a+y)-\sin y.\cos(a+y)]\frac{dy}{dx} = \cos^2(a+y)$ $\Rightarrow \sin(a+y-y)\frac{dy}{dx} = \cos^2(a+y)$ $\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$	1 1 1 1
16	Figure. Let *r* be the radius and *h* be the height of the surface of water at time *t*. Let V the volume of water in funnel $V = \frac{1}{3}\pi r^2 h$.........(1), $\frac{r}{h} = \frac{5}{10} \Rightarrow r = \frac{1}{2}h,\ \therefore \Rightarrow V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$.....(2) $\frac{dV}{dt} = -5,\ (2) \Rightarrow \frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi h^3}{12}\right) = \frac{\pi h^2}{4}\frac{dh}{dt} = -5 \Rightarrow \frac{dh}{dt} = -\frac{20}{\pi h^2}$	½ 1 1

	Rate of change of water level w.r.t time $= -\frac{20}{\pi h^2}$ When water level is 2.5cm from the top, h = 10 – 2.5 =7.5 Rate change of water level w.r.t , when h is 7.5 = $-\frac{20}{\pi(7.5)^2} = -\frac{16}{45\pi} cm/\sec$ $\therefore$ Rate of dropping of water level w.r.t when h is 7.5 = $\frac{16}{45\pi} cm/\sec.$	½ 1
OR	Slope of $y-4x+5=0$ is 4. $x^2+3y=3 \Rightarrow y=1-\frac{x^2}{3}. \therefore \frac{dy}{dx}=-\frac{2}{3}x$ Since tangent is parallel to the given line , we have $-\frac{2}{3}x=4 \Rightarrow x=-6 \Rightarrow y=-11$ The point of contact is (-6, -11) $\therefore$ the equation of the tangent is $y-(-11)=4(x-(-)) \Rightarrow 4x-y+13=0$	½ 1 1 ½ 1
17	$f(x)=\tan^{-1}(\sin x+\cos x) \Rightarrow f^1(x)=\frac{1}{1+(\sin x+\cos x)^2}(\cos x-\sin x)$ $=\frac{\cos x-\sin x}{2+\sin 2x} \Rightarrow 2+\sin 2x>0 \forall x \in (0,\pi/4)$ $\Rightarrow f^1(x)>0 \ if \cos x-\sin x>0 \Rightarrow f^1(x)>0 \ if \cos x>\sin x \ or \ \cot x>0$ Now $f^1(x)>0 \ if \tan x<1 \ if \ 0<x<\pi/4$ $\therefore f^1(x)>0 \ in \ (0.\pi/4)$ $\therefore f$ is strictly increasing function in $(0,\pi/4)$	1 1 1 1
18	$\text{I}=\int \sin^3(2x)dx \Rightarrow \int \sin^2(2x)\sin(2x)dx \Rightarrow \int [1-\cos^2(2x)]\sin(2x)dx$ Put $\cos(2x)=z \Rightarrow \sin(2x)dx=-\frac{dz}{2}$ $\text{I}=\int (1-z^2)(-\frac{dz}{2})=\frac{1}{2}\int (z^2-1)dz=\frac{1}{2}\left[\frac{1}{3}z^3-z\right]+C$ $\Rightarrow \text{I}=\frac{1}{2}\left[\frac{1}{3}\cos^3(2x)-\cos(2x)\right]+C \Rightarrow \text{I}=\frac{1}{6}\cos^3(2x)-\frac{1}{2}\cos(2x)+C$	1 1 1 1
19.	The given differential equation can be written as $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$. This is of the form $\frac{dx}{dy}+P_1x=Q_1$ where $P_1=\frac{1}{1+y^2} \ and \ Q_1=\frac{\tan^{-1}y}{1+y^2}$ $\Rightarrow I.F=e^{\int \frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$ $\Rightarrow xe^{\tan^{-1}y}=\int \left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy+C$ Let $\tan^{-1}y=t$. So that $\left(\frac{1}{1+y^2}\right)dy=dt$	1 1 1 1

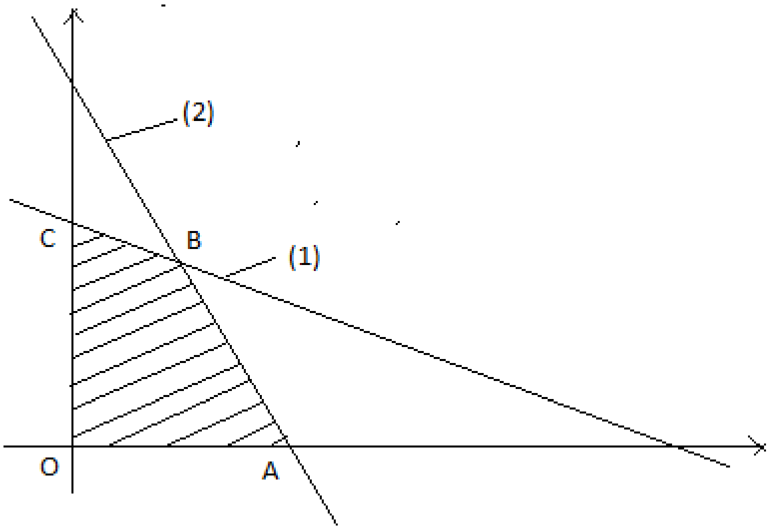
	$\Rightarrow I=\int te^t dt = te^t - \int 1.e^t dt = te^t - e^t = e^t(t-1)$ $\Rightarrow I = e^{\tan^{-1}y}(\tan^{-1}y-1) \Rightarrow x.e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y-1)+C$ $\Rightarrow x = (\tan^{-1}y-1)+Ce^{-\tan^{-1}y}$ is the general solution.	1 1
OR	The given differential equation can be written as $\frac{dy}{dx}+\left(\frac{1}{x\log x}\right)y=\frac{2}{x^2}$. This is of the form $\frac{dy}{dx}+f(x)y=g(x)$ $\Rightarrow I.F = e^{\int\frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$ $\Rightarrow y\log x = \int\frac{2\log x dx}{x^2} = \int(\log x)(2x^{-2})dx$ $\Rightarrow y\log x = \frac{-2\log x}{x}+\int 2x^{-2}dx \Rightarrow y\log x = \frac{-2\log x}{x}-\frac{2}{x}+C$ $\Rightarrow y\log x = \frac{-2}{x}(1+\log x)+C$	1 1 1 1
20	(i) $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 4\\ 2 & -5 & 2\end{vmatrix} = \hat{i}(2+20)-\hat{j}(4-8)+\hat{k}(-10-2)$ $\Rightarrow \vec{a}\times\vec{b} = 22\hat{i}+4\hat{j}-12\hat{k}$ (ii) $\vec{a}.(\vec{a}\times\vec{b}) = (2\hat{i}+\hat{j}+4\hat{k}).(22\hat{i}+4\hat{j}-12\hat{k}) = 44+4-48=0$ $\Rightarrow \vec{a}\perp(\vec{a}\times\vec{b})$ $\vec{b}.(\vec{a}\times\vec{b}) = (2\hat{i}-5\hat{j}+2\hat{k}).(22\hat{i}+4\hat{j}-12\hat{k}) = 44-20-24=0 \Rightarrow \vec{b}\perp(\vec{a}\times\vec{b})$	2 1 1
21	Equation of the plane through the point (-1,3,2) is $a(x+1)+b(y-3)+c(z-2)=0$ ……….(1) a, b,c are direction ratios of normal to the plane (1) is perpendicular to the plane , $x+2y+3z=5$ $\therefore a.1+b.2+c.3=0 \Rightarrow a+2b+3c=0$ ……(2) Also (1) is perpendicular to the plane $3x+3y+z=0$ $\therefore a.3+b.3+c.1=0 \Rightarrow 3a+3b+c=0$ ……..(3) From (2) and (3) $\frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=k \Rightarrow a=-7k, b=8k, c=-3k$ $\therefore (1) \Rightarrow -7k(x+1)+8k(y-2)-3k(z-2)=0$ $\Rightarrow 7x-8y+3z+25=0$	1 ½ ½ 1 1
22	$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{3}$ and $P(E_3)=\frac{1}{6}$ Let A is to draw a white ball. $P(A/E_1)=\frac{3}{8}, P(A/E_2)=\frac{5}{8}, P(A/E_3)=\frac{4}{8}$ $P(A)=P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)$ $=\frac{9+10+4}{48}=\frac{23}{48}$	½ 1 ½ 1 1

23	Let X denotes the number of kings. X is a random variable which takes values 0,1 or 2 $P(X=0)=\frac{^{48}C_2}{^{52}C_2}=\frac{188}{221},\ P(X=1)=\frac{^{4}C_1\,^{48}C_1}{^{52}C_2}=\frac{32}{221}$ $P(X=2)=\frac{^{4}C_2}{^{52}C_2}=\frac{1}{221}$ Mean of X = $E(X)=\sum_{i=1}^{n}x_i\,p(x_i)=0\times\frac{188}{221}+1\times\frac{1}{221}+2\times\frac{1}{221}=\frac{34}{221}$	1+1 1 1
SECTION.D		
24	*i)* f is one-one Let $x_1,x_2\in R-\{3/5\}$ and $f(x_1)=f(x_2)$ $\Rightarrow\frac{3x_1+2}{5x_1-3}=\frac{3x_2+2}{5x_2-3}\Rightarrow x_1=x_2\Rightarrow f$ is one – one *ii)* *f* is onto Let $y\in R-\{3/5\}$ $\Rightarrow y=\frac{3x+2}{5x-3}\Rightarrow x=\frac{3y+2}{5y-3}$ $\Rightarrow f\left(\frac{3y+2}{5y-3}\right)=x\Rightarrow f$ is onto. $\Rightarrow f$ is invertible. $\therefore f^{-1}$ exist. Define $g:R-\{3/5\}\to R-\{3/5\}$ g(y) = m $gof(x)=g(f(x))=g\left(\frac{3x+2}{5x-3}\right)=x\Rightarrow gof=I_{R-\{3/5\}}$ $fog(y)=f(g(y))=f\left(\frac{3y+2}{5y-3}\right)=y\Rightarrow fog=I_{R-\{3/5\}}$ $\therefore f^{-1}=g$ which is same as *f*. $\therefore f^{-1}=f$	1 1 1 ½ 1 1 ½
OR	i) f is one-one: Let $m,n\in N\cup\{0\}$ and $f(m)=f(n)\ldots\ldots(1)$ Case.1:m , n are even $(1)\Rightarrow m+1=n+1\Rightarrow m=n$ Case.2:m ,n are odd $(1)\Rightarrow m+1=n+1\Rightarrow m=n$ Case.iii: m is even and n is odd $(1)\Rightarrow m+1=n-1\Rightarrow n-m=2$ is not possible Case.iv: m is odd and n is even $(1)\Rightarrow m-1=n+1\Rightarrow m-n=2$ is not possible $\therefore f(m)=f(n)\Rightarrow m=n\Rightarrow$ f is one-one *i)* f is onto let $y\in N\cup\{0\}$ y is even $\Rightarrow y+1\in N\cup\{0\}$ and $f(y+1)=(y+1)-1=y$ y is odd $\Rightarrow y-1\in N\cup\{0\}$ and $f(y-1)=(y-1)+1=y$ $\therefore f$ is onto.	½ ½ ½ ½ ½ ½

	$\therefore f$ is invertible. $\Rightarrow$ Inverse of f exist.	
	Define $g : N \cup \{0\} \to N \cup \{0\}$ as	
	$g(m) = \begin{cases} m+1 \text{ if } m \text{ is even} \\ m-1 \text{ if } m \text{ is odd} \end{cases}$	1
	When n is odd, $gof(n) = g(f(n)) = g(n-1) = n-1+1 = n$	
	When n is even, $gof(n) = g(f(n)) = g(n+1) = n+1-1 = n$	
	When m is odd, $fog(m) = f(g(m)) = f(m-1) = m-1+1 = m$	1
	When m is even, $fog(m) = f(g(m)) = f(m+1) = m+1-1 = m$	
	$\Rightarrow gof = I_{N\cup\{0\}}$ and $fog = I_{N\cup\{0\}} \Rightarrow f^{-1} = g$ which is same as f. $\therefore f^{-1} = f$	1
		1
	$x^2 + y^2 = 4$ …….(1) $(x-2)^2 + y^2 = 4$ ……….(2). By solving (1) and (2)	1
25	we get, $A = (1, \sqrt{3}), C = (2,0), A^1 = (1, -\sqrt{3})$.	
	Required area of the enclosed region OACA'O between circles	
	= 2 [area of the region ODCAO]	1
	= 2 [area of the region ODAO + area of the region DCAD]	
	$= 2\left[\int_0^1 y dx + \int_1^2 y dx\right]$	
	$= 2\left[\int_0^1 \sqrt{4-(x-2)^2}\, dx + \int_1^2 \sqrt{4-x^2}\, dx\right]$	1
	$= 2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2} + \frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1$	
	$+2\left[\frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$	1
	$= \left[-\sqrt{3} + 4\sin^{-1}\left(\frac{1}{2}\right) - 4\sin^{-1}(-1)\right] + \left[4\sin^{-1}1 - \sqrt{3} - 4\sin^{-1}\frac{1}{2}\right]$	
	$= \left(-\sqrt{3} - \frac{2\pi}{3} + 2\pi\right) + \left(2\pi - \sqrt{3} - \frac{2\pi}{3}\right) = \frac{8\pi}{3} - 2\sqrt{3}$	1

26	$n_1 = \hat{i} + \hat{j} + \hat{k}$, $n_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $d_1 = 6$, $d_2 = -5$ Using the relation $\vec{r}.(\hat{n}_1 + \lambda\hat{n}_2) = d_1 + \lambda d_2$ we get $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) = 6 - 5\lambda$ $\Rightarrow \vec{r}.[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] = 6 - 5\lambda$ ....(1) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get $\therefore (x\hat{i} + y\hat{j} + z\hat{k}).[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] = 6 - 5\lambda$ $\Rightarrow (x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$ ....(2) Since (2) passes through (1,1,1) we have $\Rightarrow \lambda = 3/14$ Put $\lambda = 3/14$ in (1), $\Rightarrow \vec{r}.\left[(1+\frac{3}{7})\hat{i} + (1+\frac{9}{14})\hat{j} + (1+\frac{6}{7})\hat{k}\right] = 6 - \frac{15}{14}$ $\Rightarrow \hat{r}.(20\hat{i} + 23\hat{j} + 26\hat{k}) = 69$ which is the required vector equation.	1 1 1 1 1 1
27	Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$ $\therefore AX = B$ $\lvert A \rvert = 18 \neq 0$ System has unique solution $X = A^{-1}B$ adj A = $\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$ $\therefore A^{-1} = \frac{adj}{\lvert A \rvert} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ -6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$ $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ -6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$ $= \frac{1}{18}\begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $\therefore x = 2,\ y = 1,\ z = 3$	 1 1+1 1 1+1
OR	$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ $\therefore \lvert A \rvert = 67 \neq 0$ A^{-1} exist and $A^{-1} = \frac{adjA}{\lvert A \rvert} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ We have $AX = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$ $\lvert A \rvert \neq 0 \Rightarrow$ system has unique solution.: $X = A^{-1}B$	1 1+1 1

	$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ $\therefore x = 3, y = -2\ ,\ z = 1$	1 1
28	Let $I = \int_0^{\pi} \frac{xdx}{1+\sin x} \Rightarrow I = \int_0^{\pi} \frac{(\pi - x)dx}{1+\sin(\pi - x)}$ $\Rightarrow I = \int_0^{\pi} \frac{\pi dx}{1+\sin x} - \int_0^{\pi} \frac{xdx}{1+\sin x} = \pi\int_0^{\pi} \frac{dx}{1+\sin x} - I$ $\Rightarrow 2I = \pi\int_0^{\pi} \frac{dx}{1+\sin x} \Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{dx}{1+\sin x} = \frac{\pi}{2}\int_0^{\pi} \frac{1-\sin x}{1+\sin^2 x}dx$ $= \frac{\pi}{2}\int_0^{\pi} \frac{1-\sin x}{\cos^2 x}dx = \frac{\pi}{2}\int_2^{\pi}\left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos x \cos x}\right)dx$ $= \frac{\pi}{2}\int_0^{\pi}(\sec^2 x - \sec x \tan x)dx = \frac{\pi}{2}[\tan x - \sec x]_0^{\pi}$ $= \frac{\pi}{2}[\tan\pi - \sec\pi] - \frac{\pi}{2}[\tan 0 - \sec 0] = \pi$	1 1 1 1 1 1
OR	Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}dx$ ………(1) $\Rightarrow I = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}dx$ $\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}dx$……….(2) $(1)+(2) \Rightarrow 2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x}dx \Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx$ $\Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \frac{dx}{\sqrt{2(\sin x \sin\pi/4 + \cos x \cos\pi/4)}}$ $\Rightarrow \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \frac{dx}{\cos(x - \pi/4)} = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \sec(x - \pi/4)dx$ $= \frac{1}{2\sqrt{2}}\left[\log\|\sec(x - \pi/4) + \tan(x - \pi/4)\|\right]_0^{\pi/2}$ $= \frac{1}{2\sqrt{2}}\left[\log\|\sqrt{2}+1\| - \log\|\sqrt{2}-1\|\right] = \frac{1}{2\sqrt{2}}\log\frac{\sqrt{2}+1}{\sqrt{2}-1}$ $= \frac{1}{2\sqrt{2}}\log\frac{(\sqrt{2}+1)^2}{1} = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$	1 1 1 1 1 1
29	Let x be the number of goods type X and y be the number of goods type Y $\therefore x \geq 0, y \geq 0$ 30 units of labour are available. $\therefore 2x + 3y \leq 30$ ………(1) 17 units of capital are available. $\therefore 3x + y \leq 17$ ………..(2) Maximise $Z = 100x + 120y$ subject to the constraints:	½ ½ 1

	$2x + 3y \le 30$	
	$3x + y \le 17$	
	$x, y \ge 0$	
		1 1
	Vertices of the feasible region are A = (17/3,0) , B = (3,8) C = (0 ,10)	
	At O(0,0) , Z = 100(0) + 120(0) = 0	
	At A(17/3,0) , Z = 100(17/3) + 120(0) = 566.67	
	At B (3,8) , Z = 100(3) + 120 (8) = 1260	1
	At C (0, 10) , Z = 100(0) + 120(10) =1200	1
	Maximum value of Z =Rupees 1260 and occurs when x = 3 and y = 8	
	Value: Any one value	