SET – 2
Subject Code: 041

COMMON PRE-BOARD EXAMINATION 2017-2018
MATHEMATICS
MARKING SCHEME

CLASS XII

Time Allowed: 3 hours Maximum Marks: 100

Sr.No	Answer	Marks
	SECTION.A	
1.	$\Delta=\frac{1}{2}\begin{vmatrix}2&4&1\\0&1&1\\4&7&1\end{vmatrix}=\frac{1}{2}\left[2(1-7)-4(0-4)+1(0-4)\right]=\frac{1}{2}\times 0=0$ $\therefore$ Three points are collinear.	1
2.	$y=\sin^{-1}\left(\frac{2\tan\sqrt{x}}{1+\tan^2\sqrt{x}}\right)\Rightarrow y=\sin^{-1}\left[\sin 2\sqrt{x}\right]=2\sqrt{x}$ $\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{x}}$	½ ½
3.	$\frac{dy}{y^2}=-4x\,dx\Rightarrow\int\frac{dy}{y^2}=-4\int x\,dx\Rightarrow-\frac{1}{y}=-2x^2+C$ When y=1, x = 0 then C = -1. $\Rightarrow y=\frac{1}{2x^2+1}$	1
4.	Let $\vec{r}=4\hat{i}-\hat{j}+6\hat{k}\Rightarrow\lvert\vec{r}\rvert=\sqrt{4^2+(-1)^2+6^2}=\sqrt{63}$ $l=\frac{4}{\sqrt{63}},m=\frac{-1}{\sqrt{63}},n=\frac{6}{\sqrt{63}}$	½ + ½
	SECTION.B	
5	Put $x=\cos\theta\Rightarrow\sin^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{2}\right]=\sin^{-1}\left[\frac{\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}{2}\right]$ $=\sin^{-1}\left[\sin\frac{\theta}{2}\cos\frac{\pi}{4}+\cos\frac{\theta}{2}\sin\frac{\pi}{4}\right]\Rightarrow\sin^{-1}\left[\sin\left(\theta+\frac{\pi}{4}\right)\Rightarrow\theta+\frac{\pi}{4}\Rightarrow\frac{\pi}{4}+\cos^{-1}x\right]$	1 1
6	Put $x=\cos\theta$. Then $\sin^{-1}x=\theta$ $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=\sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$ $\Rightarrow\sin^{-1}(2\sin\theta\cos\theta)=\sin^{-1}(\sin 2\theta)=2\theta=2\sin^{-1}x$	1 1
7	$\begin{bmatrix}x^2\\y^2\end{bmatrix}-\begin{bmatrix}3x\\6y\end{bmatrix}=\begin{bmatrix}-2\\-9\end{bmatrix}\Rightarrow\begin{bmatrix}x^2-3x\\y^2-6y\end{bmatrix}=\begin{bmatrix}-2\\-9\end{bmatrix}\Rightarrow x^2-3x+2=0,\ y^2-6y+9=0$	½+ ½ 1

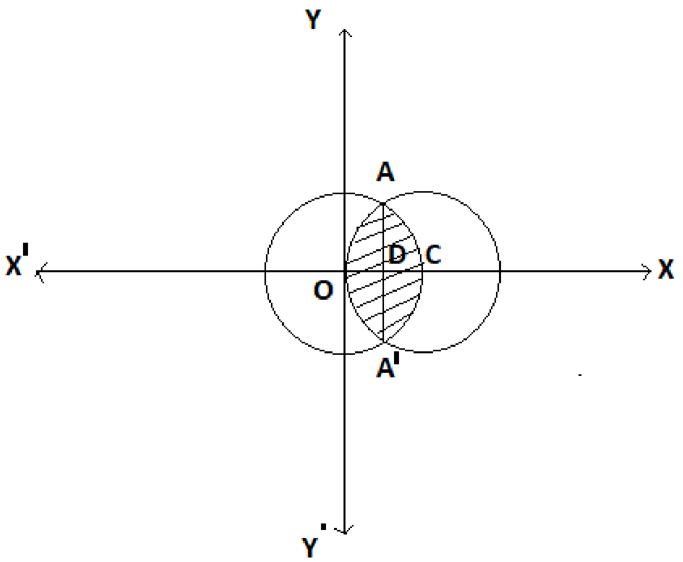
	$\Rightarrow (x-2)(x-1)=0,\ (y-3)^2=0 \Rightarrow x=1,2$ and $y=3$	
8	$y=3x^4-4x \Rightarrow \frac{dy}{dx}=12x^3-4 \Rightarrow \left.\frac{dy}{dx}\right\|_{x=4}=764$ $\therefore$ Slope of tangent = 764, Slope of normal = -1/764	½ + ½ ½+ ½
9	$\text{I}=\int\frac{dx}{4-2x-x^2}=\int\frac{dx}{-(x^2+2x-4)}=\int\frac{dx}{-[(x+1)^2-5]}$ $\Rightarrow \int\frac{dx}{(\sqrt{5})^2-(x+1)^2}=\frac{1}{2\sqrt{5}}\log\left\|\frac{\sqrt{5}+(x+1)}{\sqrt{5}-(x+1)}\right\|+C$	½+ ½ 1
10	$\frac{dy}{dx}=\frac{2\sin^2 2x}{2\cos^2 2x}=\tan^2 x \Rightarrow dy=\tan^2 x dx$ $\int dy=\int(\sec^2 2x-1)dx \Rightarrow y=\frac{1}{2}\tan 2x-x+C$	1 1
11	$\vec{r}=2\hat{a}-\hat{b}+3\hat{c} \Rightarrow \vec{r}=5\hat{i}+15\hat{j}+18\hat{k}$ $\|\vec{r}\|=\sqrt{5^2+15^2+18^2}=\sqrt{574}$units $\Rightarrow \hat{r}=\pm\frac{\vec{r}}{\|\vec{r}\|}=\pm\left[\frac{5\hat{i}+15\hat{j}+18\hat{k}}{\sqrt{574}}\right]$	½ ½ 1
12	$n(\text{S})=36$, E = Doublets .n(E) =6, F = Total of two numbers is 10, n(F) = 3 $P(F)=3/36$, $\text{n}(\text{E}\cap\text{F})=1 \Rightarrow \text{P}(\text{E}\cap\text{F})=1/36$ $\therefore P(\text{E}\|\text{F})=\frac{\text{P}(\text{E}\cap\text{F})}{\text{P(F)}}=\frac{1}{3}$	 ½ ½ 1
SECTION.C		
13	$\begin{vmatrix}-a^2 & ab & ac\\ ba & -b^2 & bc\\ ac & bc & -c^2\end{vmatrix} \Rightarrow abc\begin{vmatrix}-a & b & c\\ a & -b & c\\ a & b & -c\end{vmatrix}$ (Taking a, b, c from R_1, R_2 and R_3) $\Rightarrow (abc)(abc)\begin{vmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}$ (Taking a,b,c common from $\text{C}_1,\text{C}_2,\text{C}_3$) $=a^2b^2c^2\begin{vmatrix}-1 & 0 & 0\\ 1 & 0 & 2\\ 1 & 2 & 0\end{vmatrix}$ $(C_2\to C_1+C_2,\ C_3\to C_3+C_1)$ $\Rightarrow (a^2b^2c^2)\left[-1\begin{vmatrix}0 & 2\\ 2 & 0\end{vmatrix}-0+0\right]=4a^2b^2c^2$	1 1 1 1
14	$\lim_{x\to 1^-} f(x)=\lim_{x-1^-}(5ax-2b)=5a-2b$	1

	$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(3ax+b) = 3a+b$ F(x) is continous at x = 1, then we have $\lim_{x\to 1} f(x) = f(1)$ $\Rightarrow 5a-2b=11,\ 3a+b=11$. By solving we get a = 3 and b = 2	1 1+1
OR	$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{2} = \lim_{x\to 2}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{2}\times\frac{\sqrt{5x+2}+\sqrt{4x+4}}{\sqrt{5x+2}+\sqrt{4x+4}}$ $\Rightarrow \lim_{x\to 2}\frac{x-2}{(x-2)(\sqrt{5x+2}+\sqrt{4x+4})}$ $\Rightarrow \lim_{x\to 2}\frac{1}{\sqrt{5x+2}+\sqrt{4x+4}} = \frac{1}{\sqrt{12}+\sqrt{12}} = \frac{1}{4\sqrt{3}}$ Also f(2) = k. let f(x) be continous at x = 2. $\lim_{x\to 2} f(x) = f(2) \Rightarrow k = \frac{1}{4\sqrt{3}}$	1 1 1 1
15	Given that $\cos y = x\cos(a+y) \Rightarrow \frac{d}{dx}[\cos y] = \frac{d}{dx}[x\cos(a+y)]$ $\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y).\frac{d}{dx}(x) + x.\frac{d}{dx}[\cos(a+y)]$ $\Rightarrow --\sin y\frac{dy}{dx} = \cos(a+y) + x.[-\sin(a+y)]\frac{dy}{dx}$ $\Rightarrow [x\sin(a+y)-\sin y]\frac{dy}{dx} = \cos(a+y)$ ……….(1) $(1) \Rightarrow \left[\frac{\cos y}{\cos(a+y)}.\sin(a+y)-\sin y\right]\frac{dy}{dx} = \cos(a+y)$ $\Rightarrow [\cos y.\sin(a+y)-\sin y.\cos(a+y)]\frac{dy}{dx} = \cos^2(a+y)$ $\Rightarrow \sin(a+y-y)\frac{dy}{dx} = \cos^2(a+y)$ $\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$	1 1 1 1
16	Figure. Let *r* be the radius and *h* be the height of the surface of water at time *t*. Let V the volume of water in funnel $V = \frac{1}{3}\pi r^2 h$.........(1), $\frac{r}{h} = \frac{5}{10} \Rightarrow r = \frac{1}{2}h,\ \therefore \Rightarrow V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$.....(2) $\frac{dV}{dt} = -5,\ (2) \Rightarrow \frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi h^3}{12}\right) = \frac{\pi h^2}{4}\frac{dh}{dt} = -5 \Rightarrow \frac{dh}{dt} = -\frac{20}{\pi h^2}$ Rate of change of water level w.r.t time $= -\frac{20}{\pi h^2}$ When water level is 2.5cm from the top, h = 10 – 2.5 =7.5	½ 1 1 ½

	Rate change of water level w.r.t , when h is 7.5 = $-\frac{20}{\pi(7.5)^2} = -\frac{16}{45\pi} cm/\sec$ $\therefore$ Rate of dropping of water level w.r.t when h is 7.5 = $\frac{16}{45\pi} cm/\sec.$	1
OR	Slope of $y-4x+5=0$ is 4. $x^2+3y=3 \Rightarrow y=1-\frac{x^2}{3}$. $\therefore \frac{dy}{dx}=-\frac{2}{3}x$ Since tangent is parallel to the given line , we have $-\frac{2}{3}x=4 \Rightarrow x=-6 \Rightarrow y=-11$ The point of contact is (-6, -11) $\therefore$ the equation of the tangent is $y-(-11)=4(x-(-)) \Rightarrow 4x-y+13=0$	½ 1 1 ½ 1
17	$f(x)=\tan^{-1}(\sin x+\cos x) \Rightarrow f^1(x)=\frac{1}{1+(\sin x+\cos x)^2}(\cos x-\sin x)$ $=\frac{\cos x-\sin x}{2+\sin 2x} \Rightarrow 2+\sin 2x>0 \forall x \in (0,\pi/4)$ $\Rightarrow f^1(x)>0 \ if \cos x-\sin x>0 \Rightarrow f^1(x)>0 \ if \cos x>\sin x \ or \cot x>0$ Now $f^1(x)>0 \ if \tan x<1 \ if \ 0<x<\pi/4$ $\therefore f^1(x)>0 \ in \ (0.\pi/4)$ $\therefore f$ is strictly increasing function in $(0,\pi/4)$	1 1 1 1
18	$I=\int\frac{e^x dx}{\sqrt{5-4e^x-e^{2x}}}$, Put $z=e^x \Rightarrow dz=e^x dx$ $\Rightarrow I=\int\frac{dz}{\sqrt{5-4z-z^2}}=\int\frac{dz}{\sqrt{5-(z^2+4z)}}=\int\frac{dz}{\sqrt{5-(z^2+4z+4)+4}}$ $\Rightarrow \int\frac{dz}{\sqrt{9-(z+2)^2}}$. Let $t=z+2 \Rightarrow dt=dz$ $\Rightarrow I=\int\frac{dt}{\sqrt{3^2-t^2}}=\sin^{-1}\frac{t}{3}+C=\sin^{-1}\frac{z+2}{3}+C=\sin^{-1}\frac{e^x+2}{3}+C$	1 1 1 1
19.	The given differential equation can be written as $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$. This is of the form $\frac{dx}{dy}+P_1x=Q_1$ where $P_1=\frac{1}{1+y^2} \ and \ Q_1=\frac{\tan^{-1}y}{1+y^2}$ $\Rightarrow I.F=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$ $\Rightarrow xe^{\tan^{-1}y}=\int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy+C$ Let $\tan^{-1}y=t$. So that $\left(\frac{1}{1+y^2}\right)dy=dt$	1 1 1

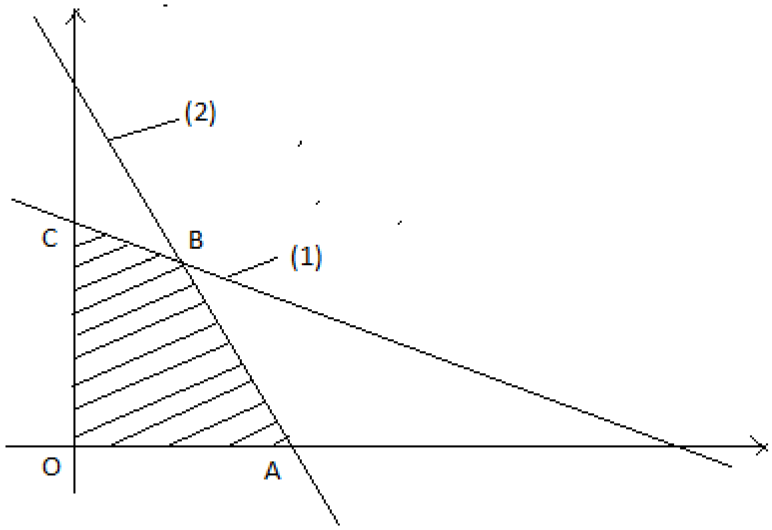
	$\Rightarrow I=\int te^t dt = te^t - \int 1.e^t dt = te^t - e^t = e^t(t-1)$ $\Rightarrow I = e^{\tan^{-1}y}(\tan^{-1}y-1) \Rightarrow x.e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y-1)+C$ $\Rightarrow x = \left(\tan^{-1}y-1\right)+Ce^{-\tan^{-1}y}$ is the general solution.	1
OR	The given differential equation can be written as $\frac{dy}{dx}+\left(\frac{1}{x\log x}\right)y=\frac{2}{x^2}$. This is of the form $\frac{dy}{dx}+f(x)y=g(x)$ $\Rightarrow I.F = e^{\int\frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$ $\Rightarrow y\log x = \int\frac{2\log x dx}{x^2} = \int(\log x)(2x^{-2})dx$ $\Rightarrow y\log x = \frac{-2\log x}{x}+\int 2x^{-2}dx \Rightarrow y\log x = \frac{-2\log x}{x}-\frac{2}{x}+C$ $\Rightarrow y\log x = \frac{-2}{x}(1+\log x)+C$	1 1 1 1
20	$\overrightarrow{AB}=\vec{b}-\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\overrightarrow{AC}=c-\vec{a}=3\hat{i}-3\hat{j}-\hat{k}$ Area of ΔABC $=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|$ $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 3 & -3 & -1\end{vmatrix}=5\hat{i}+4\hat{j}+3\hat{k}$ $\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=5\sqrt{2} \Rightarrow$ Area of ΔABC $=\frac{1}{2}\times5\sqrt{2}=\frac{5\sqrt{2}}{2}$ sq.units.	1 2 1
21	Equation of the plane through the point (-1,3,2) is $a(x+1)+b(y-3)+c(z-2)=0$ ……….(1) a, b,c are direction ratios of normal to the plane (1) is perpendicular to the plane , $x+2y+3z=5$ $\therefore a.1+b.2+c.3=0 \Rightarrow a+2b+3c=0$ ……(2) Also (1) is perpendicular to the plane $3x+3y+z=0$ $\therefore a.3+b.3+c.1=0 \Rightarrow 3a+3b+c=0$ ……..(3) From (2) and (3) $\frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=k \Rightarrow a=-7k, b=8k, c=-3k$ $\therefore$ (1) $\Rightarrow -7k(x+1)+8k(y-2)-3k(z-2)=0$ $\Rightarrow 7x-8y+3z+25=0$	1 ½ ½ 1 1
22	$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{3}$ and $P(E_3)=\frac{1}{6}$ Let A is to draw a white ball. $P(A/E_1)=\frac{3}{8}, P(A/E_2)=\frac{5}{8}, P(A/E_3)=\frac{4}{8}$ $P(A)=P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)$ $=\frac{9+10+4}{48}=\frac{23}{48}$	½ 1 ½ 1 1

23	Let X denote the number of heads in 10 trials. $n=10,\ p=1/2$ $p=1/2 \Rightarrow q=1-p \Rightarrow q=1-1/2=1/2$ $P(X=x)={}^{10}C_x\left(\frac{1}{2}\right)^{10-x}\left(\frac{1}{2}\right)^{x},\ x=0,1,2....10$ i) $P(X=6)={}^{10}C_6\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{6}=\frac{105}{512}$ ii) $P(\text{at least six heads})=\mathrm{P(X}\geq 6)$ $\Rightarrow P(X=6)+P(X=7)+P(X=8)+P(X=9)+P(X=10)$ $\Rightarrow {}^{10}C_6\left(\frac{1}{2}\right)^{10}+{}^{10}C_7\left(\frac{1}{2}\right)^{10}+{}^{10}C_8\left(\frac{1}{2}\right)^{10}+{}^{10}C_9\left(\frac{1}{2}\right)^{10}+{}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$ $=\frac{193}{512}$	1 1 1 1
	SECTION.D	
24	*i)* f is one-one Let $x_1,x_2\in R-\{3/5\}$ and $f(x_1)=f(x_2)$ $\Rightarrow \frac{3x_1+2}{5x_1-3}=\frac{3x_2+2}{5x_2-3}\Rightarrow x_1=x_2\Rightarrow f$ is one – one *ii) f* is onto Let $y\in R-\{3/5\}$ $\Rightarrow y=\frac{3x+2}{5x-3}\Rightarrow x=\frac{3y+2}{5y-3}$ $\Rightarrow f\left(\frac{3y+2}{5y-3}\right)=x\Rightarrow f$ is onto. $\Rightarrow f$ is invertible. $\therefore f^{-1}$ exist. Define $g:R-\{3/5\}\to R-\{3/5\}$ g(y) = m $gof(x)=g(f(x))=g\left(\frac{3x+2}{5x-3}\right)=x\Rightarrow gof=I_{R-\{3/5\}}$ $fog(y)=f(g(y))=f\left(\frac{3y+2}{5y-3}\right)=y\Rightarrow fog=I_{R-\{3/5\}}$ $\therefore f^{-1}=g$ which is same as *f*. $\therefore f^{-1}=f$	1 1 1 ½ 1 1 ½
OR	i) f is one-one: Let $m,n\in N\cup\{0\}$ and $f(m)=f(n)$ ……(1) Case.1:m , n are even $(1)\Rightarrow m+1=n+1\Rightarrow m=n$ Case.2:m ,n are odd $(1)\Rightarrow m+1=n+1\Rightarrow m=n$ Case.iii: m is even and n is odd $(1)\Rightarrow m+1=n-1\Rightarrow n-m=2$ is not possible Case.iv: m is odd and n is even $(1)\Rightarrow m-1=n+1\Rightarrow m-n=2$ is not possible $\therefore f(m)=f(n)\Rightarrow m=n\Rightarrow$ f is one-one *i)* f is onto	1 1 1

	let $y \in N \cup \{0\}$ y is even $\Rightarrow y+1 \in N \cup \{0\}$ and $f(y+1)=(y+1)-1=y$ y is odd $\Rightarrow y-1 \in N \cup \{0\}$ and $f(y-1)=(y-1)+1=y$ $\therefore$ f is onto. $\therefore f$ is invertible. $\Rightarrow$ Inverse of f exist. Define $g: N \cup \{0\} \to N \cup \{0\}$ as $g(m)=\begin{cases} m+1 \text{ if } m \text{ is even} \\ m-1 \text{ if } m \text{ is odd}\end{cases}$ When n is odd, $gof(n)=g(f(n))=g(n-1)=n-1+1=n$ When n is even, $gof(n)=g(f(n))=g(n+1)=n+1-1=n$ When m is odd, $fog(m)=f(g(m))=f(m-1)=m-1+1=m$ When m is even, $fog(m)=f(g(m))=f(m+1)=m+1-1=m$ $\Rightarrow gof=I_{N\cup\{0\}}$ and $fog=I_{N\cup\{0\}} \Rightarrow f^{-1}=g$ which is same as f. $\therefore f^{-1}=f$	1 1 1
25	 $x^2+y^2=4$ ....(1) $(x-2)^2+y^2=4$ ....(2). By solving (1) and (2) we get, $A=(1,\sqrt{3}), C=(2,0), A^1=(1,-\sqrt{3})$. Required area of the enclosed region OACA'O between circles = 2 [area of the region ODCAO] = 2 [area of the region ODAO + area of the region DCAD] = 2 $\left[\int_0^1 ydx+\int_1^2 ydx\right]$ = 2 $\left[\int_0^1 \sqrt{4-(x-2)^2}\,dx+\int_1^2 \sqrt{4-x^2}\,dx\right]$ = 2 $\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2}+\frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1$ $+2\left[\frac{1}{2}x\sqrt{4-x^2}+\frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$	1 1 1 1 1

	$=\left[-\sqrt{3}+4\sin^{-1}\left(\frac{1}{2}\right)-4\sin^{-1}(-1)\right]+\left[4\sin^{-1}1-\sqrt{3}-4\sin^{-1}\frac{1}{2}\right]$ $=\left(-\sqrt{3}-\frac{2\pi}{3}+2\pi\right)+\left(2\pi-\sqrt{3}-\frac{2\pi}{3}\right)=\frac{8\pi}{3}-2\sqrt{3}$	1
26	Let required line be parallel to the vector $\vec{b}$ given by , $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ The position vector of the point (1,2,-4) is $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$ The equation of the line passing through (1,2,-4) and parallel to $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}\Rightarrow\vec{r}\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$.........(1) $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$.........(2) $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$........(3) The lines (1) and (2) are perpendicular to each other. $\therefore 3b_1-16b_2+7b_3=0$........(4) Also lines (1) and (3) are perpendicular to each other $\therefore 3b_1+8b_2-b_3=0$........(5) From (4) and (5) we get $\frac{b_1}{2}=\frac{b_2}{3}=\frac{b_3}{6}$. $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})\Rightarrow\vec{r}=(1+2\lambda)\hat{i}+(2+3\lambda)\hat{j}+(6\lambda-4)\hat{k}$ This is the required equation	1 1 1 1 1 1
27	Let $A=\begin{bmatrix}1&2&1\\1&0&3\\2&-3&0\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}7\\11\\1\end{bmatrix}$ $\therefore AX=B$ $\|A\|=18\neq 0$ System has unique solution $X=A^{-1}B$ adj A = $\begin{bmatrix}9&-3&6\\6&-2&-2\\-3&7&-2\end{bmatrix}$ $\therefore A^{-1}=\frac{adj}{\|A\|}=\frac{1}{18}\begin{bmatrix}9&-3&6\\-6&-2&-2\\-3&7&-2\end{bmatrix}$ $X=A^{-1}B\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{18}\begin{bmatrix}9&-3&6\\-6&-2&-2\\-3&7&-2\end{bmatrix}\begin{bmatrix}7\\11\\1\end{bmatrix}$ $=\frac{1}{18}\begin{bmatrix}36\\18\\54\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$ $\therefore x=2,\ y=1,\ z=3$	1 1+1 1 1+1
OR	$A=\begin{bmatrix}1&2&-3\\2&3&2\\3&-3&-4\end{bmatrix}$ $\therefore\|A\|=67\neq 0$	1 1+1

	A^{-1} exist and $A^{-1} = \frac{adjA}{\lvert A\rvert} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ We have $AX = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$ $\lvert A\rvert \neq 0 \Rightarrow$ system has unique solution.: $X = A^{-1}B$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ $\therefore x = 3, y = -2, z = 1$	1 1 1
28	$I = \int_0^{\pi} \frac{xdx}{a^2\cos^2 x + b^2\sin^2 x} = \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$ $\Rightarrow \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - \int_0^{\pi} \frac{xdx}{a^2\cos^2 x + b^2\sin^2 x}$ $\Rightarrow \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - I$ $\Rightarrow I = \frac{\pi}{2}.2\int_2^{\pi/2} \frac{dx}{a^2\cos^2 + b^2\sin^2 x} = \pi\int_0^{\pi/2} \frac{\sec^2 xdx}{a^2 + b^2\tan^2 x}$ Put $b\tan x = t \Rightarrow b\sec^2 dx = dt$, When $x = 0, t = 0$ and when $x = \pi/2, t \to \infty$ $I = \frac{\pi}{b}\int_0^{\infty} \frac{dt}{a^2 + t^2} = \frac{\pi}{b}.\frac{1}{a}\left[\tan^{-1}\frac{t}{a}\right]_0^{\infty} = \frac{\pi}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{2ab}$	1 1 1 1 2
OR	$I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)dx}{1+\cos(\pi - x)}$ $I = \int_0^{\pi} \frac{(\pi - x)\sin xdx}{1+\cos^2 x} = \pi\int_0^{\pi} \frac{\sin xdx}{1+\cos^2 x} - I$ $I = \frac{\pi}{2}\int_2^{\pi} \frac{\sin x}{1+\cos^2 x}dx$ Put $\cos x = t \Rightarrow -\sin xdx = dt$. When $x = 0, t = 1$ and when $x = \pi, t = -1$ $I = \frac{-\pi}{2}\int_1^{-1} \frac{dt}{1+t^2} = \pi\int_0^{1} \frac{dt}{1+t^2}$ $= \pi\left[\tan^{-1} t\right]_0^1 = \pi\left[\tan^{-1}1 - \tan^{-1}0\right] = \pi\left[\frac{\pi}{4} - 0\right] = \frac{\pi^2}{4}$	1 1 1 1 1 1
OR	Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}dx$ ....(1) $\Rightarrow I = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}dx$ $\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}dx$..........(2)	1 1

	$(1)+(2) \Rightarrow 2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx \Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx$ $\Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \frac{dx}{\sqrt{2}(\sin x \sin \pi/4 + \cos x \cos \pi/4)}$ $\Rightarrow \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \frac{dx}{\cos(x-\pi/4)} = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \sec(x-\pi/4)dx$ $= \frac{1}{2\sqrt{2}}\left[\log\lvert\sec(x-\pi/4)+\tan(x-\pi/4)\rvert\right]_0^{\pi/2}$ $= \frac{1}{2\sqrt{2}}\left[\log\lvert\sqrt{2}+1\rvert - \log\lvert\sqrt{2}-1\rvert\right] = \frac{1}{2\sqrt{2}}\log\frac{\sqrt{2}+1}{\sqrt{2}-1}$ $= \frac{1}{2\sqrt{2}}\log\frac{(\sqrt{2}+1)^2}{1} = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$	1 1 1 1
29	Let x be the number of goods type X and y be the number of goods type Y $\therefore x \ge 0, y \ge 0$ 30 units of labour are available. $\therefore 2x+3y \le 30$ ………(1) 17 units of capital are available. $\therefore 3x + y \le 17$ ………..(2) Maximise $Z = 100x + 120y$ subject to the constraints: $2x+3y \le 30$ $3x + y \le 17$ $x, y \ge 0$  Vertices of the feasible region are A = (17/3,0) , B = (3,8) C = (0 ,10) At O(0,0) , Z = 100(0) + 120(0) = 0 At A(17/3,0) , Z = 100(17/3) + 120(0) = 566.67 At B (3,8) , Z = 100(3) + 120 (8) = 1260 At C (0, 10) , Z = 100(0) + 120(10) =1200 Maximum value of Z =Rupees 1260 and occurs when x = 3 and y = 8 Value: Any one value	½ ½ 1 1 1 1 1