SET – 3

Subject Code: 041

COMMON PRE-BOARD EXAMINATION 2017-2018
MATHEMATICS
MARKING SCHEME

CLASS XII

Time Allowed: 3 hours Maximum Marks: 100

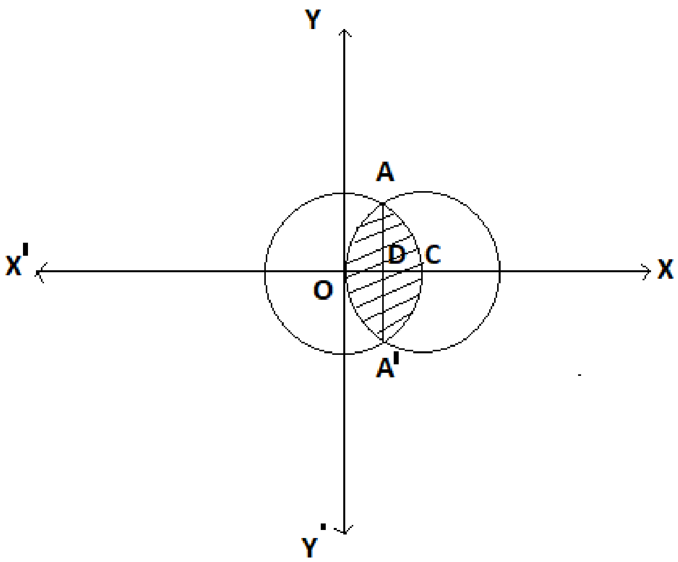
Sr.No	Answer	Marks
	SECTION.A	
1.	$y=\sin^{-1}\left(\frac{2\tan\sqrt{x}}{1+\tan^2\sqrt{x}}\right)\Rightarrow y=\sin^{-1}\left[\sin 2\sqrt{x}\right]=2\sqrt{x}$ $\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{x}}$	½ + ½
2.	$\Delta=\frac{1}{2}\begin{vmatrix}2&4&1\\0&1&1\\4&7&1\end{vmatrix}=\frac{1}{2}\left[2(1-7)-4(0-4)+1(0-4)\right]=\frac{1}{2}\times 0=0$ $\therefore$ Three points are collinear.	1
3.	$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}=\frac{3-4+10}{\sqrt{9+1+4}.\sqrt{1+16+25}}=\frac{9}{\sqrt{588}}\Rightarrow\theta=\cos^{-1}\left(\frac{9}{14\sqrt{3}}\right)$	1
4.	$\frac{dy}{y^2}=-4x\,dx\Rightarrow\int\frac{dy}{y^2}=-4\int x\,dx\Rightarrow-\frac{1}{y}=-2x^2+C$ When y=1, x = 0 then C = -1. $\Rightarrow y=\frac{1}{2x^2+1}$	½ + ½
	SECTION.B	
5	$y=3x^4-4x\Rightarrow\frac{dy}{dx}=12x^3-4\Rightarrow\frac{dy}{dx}\Big\rvert_{x=4}=764$ $\therefore$ Slope of tangent = 764, Slope of normal = -1/764	1 1
6	$\text{I}=\int\frac{dx}{4-2x-x^2}=\int\frac{dx}{-\left(x^2+2x-4\right)}=\int\frac{dx}{-\left[(x+1)^2-5\right]}$ $\Rightarrow\int\frac{dx}{\left(\sqrt{5}\right)^2-(x+1)^2}=\frac{1}{2\sqrt{5}}\log\left\lvert\frac{\sqrt{5}+(x+1)}{\sqrt{5}-(x+1)}\right\rvert+C$	1 1
7	Put $x=\cos\theta$. Then $\sin^{-1}x=\theta$ $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=\sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$ $\Rightarrow\sin^{-1}(2\sin\theta\cos\theta)=\sin^{-1}(\sin 2\theta)=2\theta=2\sin^{-1}x$	1 1

8	$x=\cos\theta \Rightarrow \sin^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{2}\right]$ $\Rightarrow \sin^{-1}\left[\frac{\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}{2}\right]$ $=\sin^{-1}\left[\sin\frac{\theta}{2}\cos\frac{\pi}{4}+\cos\frac{\theta}{2}\sin\frac{\pi}{4}\right]$ $\Rightarrow \sin^{-1}\left[\sin\left(\theta+\frac{\pi}{4}\right)\Rightarrow \theta+\frac{\pi}{4}\Rightarrow \frac{\pi}{4}+\cos^{-1}x\right]$	1 1
9	$\begin{bmatrix}x^2\\y^2\end{bmatrix}-\begin{bmatrix}3x\\6y\end{bmatrix}=\begin{bmatrix}-2\\-9\end{bmatrix}\Rightarrow\begin{bmatrix}x^2-3x\\y^2-6y\end{bmatrix}=\begin{bmatrix}-2\\-9\end{bmatrix}\Rightarrow x^2-3x+2=0,\ y^2-6y+9=0$ $\Rightarrow (x-2)(x-1)=0,\ (y-3)^2=0\Rightarrow x=1,2$ and $y=3$	1 1
10	$\frac{dx}{dy}=x+y+1\Rightarrow \frac{dx}{dy}-x=y+1\Rightarrow I.F=e^{\int -dy}=e^{-y}$ $\Rightarrow xe^{-y}=\int(y+1)e^{-y}dy\Rightarrow xe^{-y}=-(y+1)e^{-y}-e^{-y}+C\Rightarrow x=-(y+2)+Ce^{y}$	1 1
11	n(S) = 36 , E = Doublets .n(E) =6, F = Total of two numbers is 10, n(F) = 3 $P(F)=3/36$, $\text{n}(\text{E}\cap\text{F})=1\Rightarrow \text{P}(\text{E}\cap\text{F})=1/36$ $\therefore P(\text{E}\mid\text{F})=\frac{\text{P}(\text{E}\cap\text{F})}{\text{P}(\text{F})}=\frac{1}{3}$	½ ½ 1
12	$\vec{r}=2\hat{a}-\hat{b}+3\hat{c}\Rightarrow \vec{r}=5\hat{i}+15\hat{j}+18\hat{k}$ $\lvert\vec{r}\rvert=\sqrt{5^2+15^2+18^2}=\sqrt{574}$units $\Rightarrow \hat{r}=\pm\frac{\vec{r}}{\lvert\vec{r}\rvert}=\pm\left[\frac{5\hat{i}+15\hat{j}+18\hat{k}}{\sqrt{574}}\right]$	1 1
SECTION.C		
13	Given that $\cos y=x\cos(a+y)\Rightarrow \frac{d}{dx}[\cos y]=\frac{d}{dx}[x\cos(a+y)]$ $\Rightarrow -\sin y\frac{dy}{dx}=\cos(a+y).\frac{d}{dx}(x)+x.\frac{d}{dx}[\cos(a+y)]$ $\Rightarrow --\sin y\frac{dy}{dx}=\cos(a+y)+x.[-\sin(a+y)]\frac{dy}{dx}$ $\Rightarrow [x\sin(a+y)-\sin y]\frac{dy}{dx}=\cos(a+y)$ ………..(1) $(1)\Rightarrow\left[\frac{\cos y}{\cos(a+y)}.\sin(a+y)-\sin y\right]\frac{dy}{dx}=\cos(a+y)$ $\Rightarrow[\cos y.\sin(a+y)-\sin y.\cos(a+y)]\frac{dy}{dx}=\cos^2(a+y)$	1 1 1

	$\Rightarrow \sin(a+y-y)\dfrac{dy}{dx}=\cos^2(a+y)$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$	1
14	$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} \Rightarrow abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$ (Taking a, b, c from R_1, R_2 and R_3) $\Rightarrow (abc)(abc)\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ (Taking a,b,c common from C_1, C_2, C_3) $= a^2b^2c^2\begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix}$ $(C_2 \to C_1 + C_2,\ C_3 \to C_3 + C_1)$ $\Rightarrow \left(a^2b^2c^2\right)\left[-1\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} - 0 + 0\right] = 4a^2b^2c^2$	1 1 1 1
15	Figure. Let *r* be the radius and *h* be the height of the surface of water at time *t*. Let V the volume of water in funnel $V=\dfrac{1}{3}\pi r^2 h.........(1), \dfrac{r}{h}=\dfrac{5}{10} \Rightarrow r=\dfrac{1}{2}h, \ \therefore \Rightarrow V=\dfrac{1}{3}\pi\left(\dfrac{h}{2}\right)^2 h=\dfrac{\pi h^3}{12}.....(2)$ $\dfrac{dV}{dt}=-5,\ (2) \Rightarrow \dfrac{dV}{dt}=\dfrac{d}{dt}\left(\dfrac{\pi h^3}{12}\right)=\dfrac{\pi h^2}{4}\dfrac{dh}{dt}=-5 \Rightarrow \dfrac{dh}{dt}=-\dfrac{20}{\pi h^2}$ Rate of change of water level w.r.t time $=-\dfrac{20}{\pi h^2}$ When water level is 2.5cm from the top, h = 10 – 2.5 =7.5 Rate change of water level w.r.t , when h is 7.5 = $-\dfrac{20}{\pi(7.5)^2}=-\dfrac{16}{45\pi}\,cm/\sec$ $\therefore$ Rate of dropping of water level w.r.t when h is 7.5 = $\dfrac{16}{45\pi}\,cm/\sec.$	½ 1 1 ½ 1
OR	Slope of $y-4x+5=0$ is 4. $x^2+3y=3 \Rightarrow y=1-\dfrac{x^2}{3}.\ \therefore \dfrac{dy}{dx}=-\dfrac{2}{3}x$ Since tangent is parallel to the given line , we have $-\dfrac{2}{3}x=4 \Rightarrow x=-6 \Rightarrow y=-11$ The point of contact is (-6, -11) $\therefore$ the equation of the tangent is $y-(-11)=4(x-(-)) \Rightarrow 4x-y+13=0$	1 1 1 1

16	$f(x)=\tan^{-1}(\sin x+\cos x)\Rightarrow f^{1}(x)=\frac{1}{1+(\sin x+\cos x)^2}(\cos x-\sin x)$ $=\frac{\cos x-\sin x}{2+\sin 2x}\Rightarrow 2+\sin 2x>0\forall x\in(0,\pi/4)$ $\Rightarrow f^{1}(x)>0\ if\ \cos x-\sin x>0\Rightarrow f^{1}(x)>0\ if\ \cos x>\sin x\ or\ \cot x>0$ Now $f^{1}(x)>0\ if\ \tan x<1\ if\ 0<x<\pi/4$ $\therefore f^{1}(x)>0\ in\ (0.\pi/4)$ $\therefore f$ is strictly increasing function in $(0,\pi/4)$	1 1 1 1
17	$\lim_{x\to 1^-} f(x)=\lim_{x-1^-}(5ax-2b)=5a-2b$ $\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(3ax+b)=3a+b$ F(x) is continous at x = 1, then we have $\lim_{x\to 1} f(x)=f(1)$ $\Rightarrow 5a-2b=11,\ 3a+b=11$. By solving we get a = 3 and b = 2	1 1 1+1
OR	$\lim_{x\to 2} f(x)=\lim_{x\to 2}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{2}=\lim_{x\to 2}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{2}\times\frac{\sqrt{5x+2}+\sqrt{4x+4}}{\sqrt{5x+2}+\sqrt{4x+4}}$ $\Rightarrow \lim_{x\to 2}\frac{x-2}{(x-2)(\sqrt{5x+2}+\sqrt{4x+4})}$ $\Rightarrow \lim_{x\to 2}\frac{1}{\sqrt{5x+2}+\sqrt{4x+4}}=\frac{1}{\sqrt{12}+\sqrt{12}}=\frac{1}{4\sqrt{3}}$ Also f(2) = k. let f(x) be continous at x = 2. $\lim_{x\to 2} f(x)=f(2)\Rightarrow k=\frac{1}{4\sqrt{3}}$	1 1 1 1
18	Equation of the plane through the point (-1,3,2) is $a(x+1)+b(y-3)+c(z-2)=0$ ……….(1) a, b,c are direction ratios of normal to the plane (1) is perpendicular to the plane , $x+2y+3z=5$ $\therefore a.1+b.2+c.3=0\Rightarrow a+2b+3c=0$ ……(2) Also (1) is perpendicular to the plane $3x+3y+z=0$ $\therefore a.3+b.3+c.1=0\Rightarrow 3a+3b+c=0$ ……..(3) From (2) and (3) $\frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=k\Rightarrow a=-7k,b=8k,c=-3k$ $\therefore (1)\Rightarrow -7k(x+1)+8k(y-2)-3k(z-2)=0$ $\Rightarrow 7x-8y+3z+25=0$	1 1 1 1
19	$\text{I}=\int\sin^3(2x)dx\Rightarrow\int\sin^2(2x)\sin(2x)dx\Rightarrow\int[1-\cos^2(2x)]\sin(2x)dx$ Put $\cos(2x)=z\Rightarrow\sin(2x)dx=-\frac{dz}{2}$ $\text{I}=\int(1-z^2)(-\frac{dz}{2})=\frac{1}{2}\int(z^2-1)dz=\frac{1}{2}\left[\frac{1}{3}z^3-z\right]+C$	1 1 1

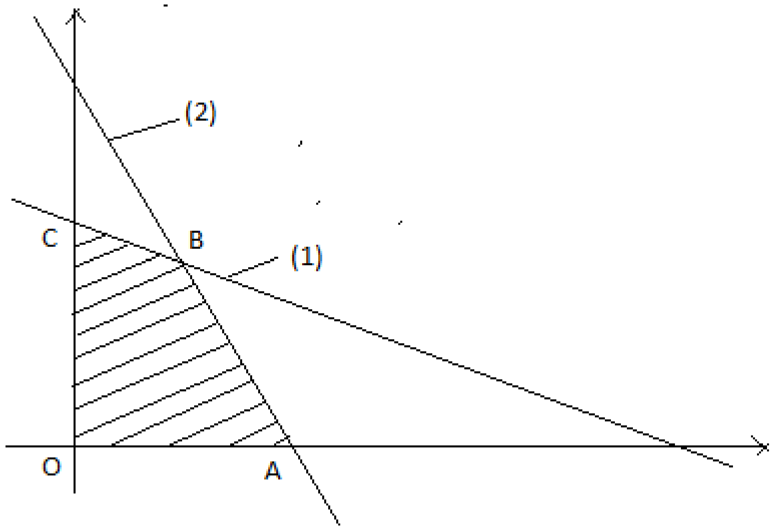
	$\Rightarrow \text{I} = \frac{1}{2}\left[\frac{1}{3}\cos^3(2x) - \cos(2x)\right] + C \Rightarrow \text{I} = \frac{1}{6}\cos^3(2x) - \frac{1}{2}\cos(2x) + C$	1
20	Let X denotes the number of kings. X is a random variable which takes values 0,1 or 2 $P(X=0) = \frac{^{48}C_2}{^{52}C_2} = \frac{188}{221},\ P(X=1) = \frac{^{4}C_1\,^{48}C_1}{^{52}C_2} = \frac{32}{221}$ $P(X=2) = \frac{^{4}C_2}{^{52}C_2} = \frac{1}{221}$ Mean of X = $E(X) = \sum_{i=1}^{n} x_i\, p(x_i) = 0 \times \frac{188}{221} + 1 \times \frac{1}{221} + 2 \times \frac{1}{221} = \frac{34}{221}$	1 1 2
21	The given differential equation can be written as $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$. This is of the form $\frac{dx}{dy} + P_1 x = Q_1$ where $P_1 = \frac{1}{1+y^2}$ and $Q_1 = \frac{\tan^{-1} y}{1+y^2}$ $\Rightarrow I.F = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$ $\Rightarrow x e^{\tan^{-1} y} = \int \left(\frac{\tan^{-1} y}{1+y^2}\right) e^{\tan^{-1} y} dy + C$ Let $\tan^{-1} y = t$. So that $\left(\frac{1}{1+y^2}\right) dy = dt$ $\Rightarrow I = \int t e^t dt = t e^t - \int 1.e^t dt = t e^t - e^t = e^t (t-1)$ $\Rightarrow I = e^{\tan^{-1} y}(\tan^{-1} y - 1) \Rightarrow x.e^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + C$ $\Rightarrow x = (\tan^{-1} y - 1) + C e^{-\tan^{-1} y}$ is the general solution.	1 1 1 1
OR	The given differential equation can be written as $\frac{dy}{dx} + \left(\frac{1}{x \log x}\right) y = \frac{2}{x^2}$. This is of the form $\frac{dy}{dx} + f(x) y = g(x)$ $\Rightarrow I.F = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$ $\Rightarrow y \log x = \int \frac{2 \log x\, dx}{x^2} = \int (\log x)(2x^{-2}) dx$ $\Rightarrow y \log x = \frac{-2 \log x}{x} + \int 2x^{-2} dx \Rightarrow y \log x = \frac{-2 \log x}{x} - \frac{2}{x} + C$ $\Rightarrow y \log x = \frac{-2}{x}(1 + \log x) + C$	1 1 1 1
22	Let X denote the number of heads in 10 trials. $n = 10,\ p = 1/2$ $p = 1/2 \Rightarrow q = 1 - p \Rightarrow q = 1 - 1/2 = 1/2$	

	$P(X=x)={}^{10}C_x\left(\frac{1}{2}\right)^{10-x}\left(\frac{1}{2}\right)^x$, $x=0,1,2....10$	1
	i) $P(X=6)={}^{10}C_6\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6=\frac{105}{512}$	1
	ii) $P(\text{at least six heads}) = P(X \geq 6)$ $\Rightarrow P(X=6)+P(X=7)+P(X=8)+P(X=9)+P(X=10)$ $\Rightarrow {}^{10}C_6\left(\frac{1}{2}\right)^{10}+{}^{10}C_7\left(\frac{1}{2}\right)^{10}+{}^{10}C_8\left(\frac{1}{2}\right)^{10}+{}^{10}C_9\left(\frac{1}{2}\right)^{10}+{}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$	1
	$=\frac{193}{512}$	1
23	$\overrightarrow{AB}=\vec{b}-\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\overrightarrow{AC}=c-\vec{a}=3\hat{i}-3\hat{j}-\hat{k}$	1
	Area of $\Delta ABC=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|$ $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 3 & -3 & -1\end{vmatrix}=5\hat{i}+4\hat{j}+3\hat{k}$	1+1
	$\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=5\sqrt{2} \Rightarrow \text{Area of } \Delta ABC=\frac{1}{2}\times 5\sqrt{2}=\frac{5\sqrt{2}}{2}$ sq.units.	1
	SECTION.D	
24		1
	$x^2+y^2=4$ …….(1) $(x-2)^2+y^2=4$ ……….(2). By solving (1) and (2) we get, $A=(1,\sqrt{3}), C=(2,0), A^1=(1,-\sqrt{3})$.	1
	Required area of the enclosed region OACA'O between circles = 2 [area of the region ODCAO] = 2 [area of the region ODAO + area of the region DCAD] = 2 $\left[\int_0^1 ydx+\int_1^2 ydx\right]$	1

	$= 2\left[\int_0^1 \sqrt{4-(x-2)^2}\,dx + \int_1^2 \sqrt{4-x^2}\,dx\right]$ $= 2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2} + \frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1$ $+2\left[\frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$ $=\left[-\sqrt{3}+4\sin^{-1}\left(\frac{1}{2}\right)-4\sin^{-1}(-1)\right]+\left[4\sin^{-1}1-\sqrt{3}-4\sin^{-1}\frac{1}{2}\right]$ $=\left(-\sqrt{3}-\frac{2\pi}{3}+2\pi\right)+\left(2\pi-\sqrt{3}-\frac{2\pi}{3}\right)=\frac{8\pi}{3}-2\sqrt{3}$	1 1 1
25	Let $A=\begin{bmatrix}1 & 2 & 1\\ 1 & 0 & 3\\ 2 & -3 & 0\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}7\\ 11\\ 1\end{bmatrix} \quad \therefore AX=B$ $\lvert A\rvert = 18 \neq 0$ System has unique solution $X=A^{-1}B$ adj A = $\begin{bmatrix}9 & -3 & 6\\ 6 & -2 & -2\\ -3 & 7 & -2\end{bmatrix} \quad \therefore A^{-1}=\frac{adj}{\lvert A\rvert}=\frac{1}{18}\begin{bmatrix}9 & -3 & 6\\ -6 & -2 & -2\\ -3 & 7 & -2\end{bmatrix}$ $X=A^{-1}B \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{18}\begin{bmatrix}9 & -3 & 6\\ -6 & -2 & -2\\ -3 & 7 & -2\end{bmatrix}\begin{bmatrix}7\\ 11\\ 1\end{bmatrix}$ $=\frac{1}{18}\begin{bmatrix}36\\ 18\\ 54\end{bmatrix}=\begin{bmatrix}2\\ 1\\ 3\end{bmatrix} \quad \therefore x=2,\ y=1,\ z=3$	1 1+1 1 1+1
OR	$A=\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix} \quad \therefore \lvert A\rvert = 67 \neq 0$ A^{-1} exist and $A^{-1}=\frac{adjA}{\lvert A\rvert}=\frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{bmatrix}$ We have AX = B, where $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\ B=\begin{bmatrix}-4\\ 2\\ 11\end{bmatrix}$ $\lvert A\rvert \neq 0 \Rightarrow$ system has unique solution.: $X=A^{-1}B$	1 1+1 1

	$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ $\therefore x = 3, y = -2\ ,\ z = 1$	1 1
26	Let required line be parallel to the vector $\vec{b}$ given by , $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ The position vector of the point (1,2,-4) is $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ The equation of the line passing through (1,2,-4) and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b} \Rightarrow \vec{r}\left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$.........(1) $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$.........(2) $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$........(3) The lines (1) and (2) are perpendicular to each other. $\therefore 3b_1 - 16b_2 + 7b_3 = 0$........(4) Also lines (1) and (3) are perpendicular to each other $\therefore 3b_1 + 8b_2 - b_3 = 0$........(5) From (4) and (5) we get $\frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \Rightarrow \vec{r} = (1 + 2\lambda)\hat{i} + (2 + 3\lambda)\hat{j} + (6\lambda - 4)\hat{k}$ This is the required equation	1 1 1 1 1 1
27	*i)* f is one-one Let $x_1, x_2 \in R - \{3/5\}$ and $f(x_1) = f(x_2)$ $\Rightarrow \frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3} \Rightarrow x_1 = x_2 \Rightarrow f$ is one – one *ii) f* is onto Let $y \in R - \{3/5\}$ $\Rightarrow y = \frac{3x+2}{5x-3} \Rightarrow x = \frac{3y+2}{5y-3}$ $\Rightarrow f\left(\frac{3y+2}{5y-3}\right) = x \Rightarrow f$ is onto. $\Rightarrow f$ is invertible. $\therefore f^{-1}$ exist. Define $g : R - \{3/5\} \to R - \{3/5\}$ g(y) = x $gof(x) = g(f(x)) = g\left(\frac{3x+2}{5x-3}\right) = x \Rightarrow gof = I_{R-\{3/5\}}$ $fog(y) = f(g(y)) = f\left(\frac{3y+2}{5y-3}\right) = y \Rightarrow fog = I_{R-\{3/5\}}$ $\therefore f^{-1} = g$ which is same as *f*. $\therefore f^{-1} = f$	1 1 1 1 1 1
OR	i) f is one-one: Let $m, n \in N \cup \{0\}$ and $f(m) = f(n)$ ……(1) Case.1:m , n are even $(1) \Rightarrow m + 1 = n + 1 \Rightarrow m = n$ Case.2:m ,n are odd $(1) \Rightarrow m + 1 = n + 1 \Rightarrow m = n$	½ ½

	Case.iii: m is even and n is odd $(1) \Rightarrow m+1 = n-1 \Rightarrow n-m = 2$ is not possible Case.iv: m is odd and n is even $(1) \Rightarrow m-1 = n+1 \Rightarrow m-n = 2$ is not possible $\therefore\ f(m) = f(n) \Rightarrow m = n \Rightarrow$ f is one-one *i)* f is onto let $y \in N \cup \{0\}$ y is even $\Rightarrow y+1 \in N \cup \{0\}$ and $f(y+1) = (y+1)-1 = y$ y is odd $\Rightarrow y-1 \in N \cup \{0\}$ and $f(y-1) = (y-1)+1 = y$ $\therefore\ f$ is onto. $\therefore f$ is invertible. $\Rightarrow$ Inverse of f exist. Define $g : N \cup \{0\} \to N \cup \{0\}$ as $g(m) = \begin{cases} m+1 \text{ if } m \text{ is even} \\ m-1 \text{ if } m \text{ is odd} \end{cases}$ When n is odd, $gof(n) = g(f(n)) = g(n-1) = n-1+1 = n$ When n is even, $gof(n) = g(f(n)) = g(n+1) = n+1-1 = n$ When m is odd, $fog(m) = f(g(m)) = f(m-1) = m-1+1 = m$ When m is even, $fog(m) = f(g(m)) = f(m+1) = m+1-1 = m$ $\Rightarrow gof = I_{N\cup\{0\}}$ and $fog = I_{N\cup\{0\}} \Rightarrow f^{-1} = g$ which is same as *f*. $\therefore\ f^{-1} = f$	½ ½ ½ ½ ½ ½ 1 1
28	$I = \int_0^{\pi} \frac{xdx}{a^2\cos^2 x + b^2 \sin^2 x} = \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$ $\Rightarrow \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - \int_0^{\pi} \frac{xdx}{a^2\cos^2 x + b^2\sin^2 x}$ $\Rightarrow \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - I$ $\Rightarrow I = \frac{\pi}{2}.2\int_2^{\pi/2} \frac{dx}{a^2\cos^2 + b^2\sin^2 x} = \pi\int_0^{\pi/2} \frac{\sec^2 xdx}{a^2 + b^2\tan^2 x}$ Put $b\tan x = t \Rightarrow b\sec^2 dx = dt$, When $x = 0, t = 0$ and when $x = \pi/2, t \to \infty$ $I = \frac{\pi}{b}\int_0^{\infty} \frac{dt}{a^2 + t^2} = \frac{\pi}{b}.\frac{1}{a}\left[\tan^{-1}\frac{t}{a}\right]_0^{\infty} = \frac{\pi}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{2ab}$	1 1 1 1 2
OR	$I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)dx}{1 + \cos(\pi - x)}$ $I = \int_0^{\pi} \frac{(\pi - x)\sin xdx}{1+\cos^2 x} = \pi\int_0^{\pi} \frac{\sin xdx}{1+\cos^2 x} - I$ $I = \frac{\pi}{2}\int_2^{\pi} \frac{\sin x}{1+\cos^2 x}dx$ Put $\cos x = t \Rightarrow -\sin xdx = dt$. When $x = 0, t = 1$ and when $x = \pi, t = -1$ $I = \frac{-\pi}{2}\int_1^{-1} \frac{dt}{1+t^2} = \pi\int_0^1 \frac{dt}{1+t^2}$	1 1 1 1 1 1

	$=\pi\left[\tan^{-1}t\right]_0^1=\pi\left[\tan^{-1}1-\tan^{-1}0\right]=\pi\left[\frac{\pi}{4}-0\right]=\frac{\pi^2}{4}$	
29	Let x be the number of goods type X and y be the number of goods type Y $\therefore x\geq 0, y\geq 0$ 30 units of labour are available. $\therefore 2x+3y\leq 30$ ....(1) 17 units of capital are available. $\therefore 3x+y\leq 17$ ....(2) Maximise $Z=100x+120y$ subject to the constraints: $2x+3y\leq 30, 3x+y\leq 17,\ x,y\geq 0$  Vertices of the feasible region are A = (17/3,0) , B = (3,8) C = (0 ,10) At O(0,0) , Z = 100(0) + 120(0) = 0 At A(17/3,0) , Z = 100(17/3) + 120(0) = 566.67 At B (3,8) , Z = 100(3) + 120 (8) = 1260 At C (0, 10) , Z = 100(0) + 120(10) =1200 Maximum value of Z =Rupees 1260 and occurs when x = 3 and y = 8 Value: Any one value	½ ½ 1 1 1 1 1