SET – 1
Subject Code: 041

COMMON PRE-BOARD EXAMINATION 2017-2018

MATHEMATICS

CLASS: XII

Time Allowed: 3 hours Maximum Marks: 100

General Instructions:

*i. **ALL** questions are compulsory.*

ii. This question paper contains 29 questions.

iii.Question 1-4 in Section Aare very short-answer type questions carrying 1 mark each.

iv. Question 5-12 in Section B are short-answer type questions carrying 2 marks each.

*v. Question 13-23 in Section C are long-answer **I** type questions carrying 4 marks each.*

*vi. Question 24-29 in Section D are long-answer **II** type questions carrying 6 marks each.*

SECTION.A

Questions 1 to 4 carry 1 mark each.

1. Using determinants, show that three points A (2, 4), B (0, 1) and C (4, 7) are collinear.

2. Find $\frac{dy}{dx}$, if $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$.

3. Find the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ given that $y = 1$ when $x = 0$

4. Find the angle between two vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$.

SECTION.B

Questions 5 to 12 carry 2 marks each.

5. Write in simplest form: $\sin^{-1}\left[\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right]$

6. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then prove that $x + y + z = xyz$.

7. Solve for x and y when: $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$

8. Find the slopes of tangent and normal to the curve $y = 3x^4 - 4x$ at $x = 4$.

9. Evaluate: $\int \frac{dx}{4-2x-x^2}$

10. Solve the differential equation: $(x+y+1)\frac{dy}{dx}=1$

11. If $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-4\hat{j}+\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}+5\hat{k}$. Find a unit vector along the vector $\vec{r}=2\hat{a}-\hat{b}+3\hat{c}$.

12. A pair of dice is thrown. Find the probability of getting a doublet if it is known that sum of the numbers on the two dice is 10.

SECTION.C

Questions 13 to 23 carry 4 marks each.

13. Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

14. If the function $f(x)=\begin{cases} 3ax+b & if\ x>1 \\ 11 & if\ x=1 \\ 5ax-2b & if\ x<1 \end{cases}$ is continuous at $x=1$, find the values of *a* and *b*.

OR

For what value of *k*, the function $f(x)=\begin{cases} \frac{\sqrt{5x+2}-\sqrt{4x+4}}{x-2}, & if\ x\neq 2 \\ k & if\ x=2 \end{cases}$ is continuous at $x=2$?

15. If $\cos y = x\cos(a+y)$, with $\cos a \neq 1$, prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$.

16. Water is leaking from a conical funnel at the rate of $5cm^3/\sec$. If the radius of the base of the funnel is 5cm and the altitude is 10cm, find the rate at which the water level is dropping when it is 2.5cm from the top.

OR

Find the equation of the tangent to the curve $x^2+3y=3$ which is parallel to the line $y-4x+5=0$.

17. Show that the function *f* given by $f(x)=\tan^{-1}(\sin x+\cos x)$, $x>0$ is always a strictly increasing function in $\left(0,\frac{\pi}{4}\right)$.

18. Evaluate: $\int \sin^3(2x)dx$

19. Solve the differential equation $\left(\tan^{-1} y - x\right)dy = \left(1 + y^2\right)dx$

OR

Solve: $\left(x \log x\right)\left(\frac{dy}{dx}\right) + y = \frac{2\log x}{x}$

20. Show that the vectors $\vec{a} = 2\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - 5\hat{j} + 2\hat{k}$, find (i) $\vec{a} \times \hat{b}$ (ii) show that $\vec{a} \perp \left(\vec{a} \times \vec{b}\right)$ and $\vec{b} \perp \left(\vec{a} \times \vec{b}\right)$

21. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

22. There are three bags, I, II and III. Bag I has 3 white, 5 black balls, bag II has 5 white, 3 black balls and bag III has 4 white, 4 black balls. A bag is selected at random and a ball is drawn from it. What is the probability that the ball drawn is white? Probabilities of the selection of bags are $\frac{1}{2}, \frac{1}{3},$ and $\frac{1}{6}$ respectively.

23. Two cards drawn simultaneously (or without replacement) from a well shuffled pack of 52 cards. Find the mean of the number of kings.

SECTION.D

Questions 24 to 29 carry 6 marks each.

24. If $f : R - \{3/5\} \rightarrow R - \{3/5\}$ be a function defined by $f(x) = \frac{3x+2}{5x-3}, x \in R - \{3/5\}$. Show that $f^{-1}(x) = f(x), x \in R - \{3/5\}$

OR

Let $f : N \cup \{0\} \rightarrow N \cup \{0\}$ be defined by $f(n) = \begin{cases} n+1, if\ n\ is\, even \\ n-1, if\ n\ is\, odd \end{cases}$. Show that f is invertible and $f = f^{-1}$

25. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $\left(x-2\right)^2 + y^2 = 4$.

26. Find vector equations of the planes passing through the intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$, and the point (1, 1, 1).

27. Solve the system of equations by matrix method: $x+2y+z=7; x+3z=11; 2x-3y=1$

OR

Find A^{-1} if $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}$. Hence solve the equations:

$x+2y-3z=-4, 2x+3y+2z=2; 3x-3y-4z=11$.

28. Evaluate: $\int_0^{\pi}\frac{x\,dx}{1+\sin x}$

OR

Evaluate: $\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$

29. A producer has 30 units of labour and 17 units of capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly 3 units of labour and 1 unit of capital are required to produce one unit of Y. If X and Y are priced at rupees 100 and rupees 120 per unit respectively, how should the producer use his resources to maximize the total revenue? Solve the problem graphically. What quality of the producer is depicted in his activity?

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