SET – 3

Subject Code: 041

COMMON PRE-BOARD EXAMINATION 2017-2018

MATHEMATICS

CLASS:XII

Time Allowed: 3 hours Maximum Marks: 100

General Instructions:

*i. **ALL** questions are compulsory.*

ii. This question paper contains 29 questions.

iii.Question 1-4 in Section Aare very short-answer type questions carrying 1 mark each.

iv. Question 5-12 in Section B are short-answer type questions carrying 2 marks each.

*v. Question 13-23 in Section C are long-answer **I** type questions carrying 4 marks each.*

*vi. Question 24-29 in Section D are long-answer **II** type questions carrying 6 marks each.*

SECTION.A

Questions 1 to 4 carry 1 mark each.

1. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{2\tan\sqrt{x}}{1+\tan^2\sqrt{x}}\right)$

2. Using determinants, show that three points A (2, 4), B (0, 1) and C (4, 7) are collinear.

3. Find the angle between two vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$.

4. Find the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ given that $y = 1$ when $x = 0$

SECTION.B

Questions 5 to 12 carry 2 marks each.

5. Find the slopes of tangent and normal to the curve $y = 3x^4 - 4x$ at $x = 4$.

6. Evaluate: $\int \frac{dx}{4 - 2x - x^2}$

7. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x, -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

8. Write in simplest form: $\sin^{-1}\left[\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right]$

9. Solve for x and y when: $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$

10. Solve the differential equation: $(x + y + 1)\dfrac{dy}{dx} = 1$

11. A pair of dice is thrown. Find the probability of getting a doublet if it is known that sum of the numbers on the two dice is 10.

12. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + 5\hat{k}$. Find a unit vector along the vector $\vec{r} = 2\hat{a} - \hat{b} + 3\hat{c}$.

SECTION.C

Questions 13 to 23 carry 4 marks each.

13. If $\cos y = x\cos(a + y)$, with $\cos a \neq 1$, prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{\sin a}$.

14. Show that $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

15. Water is leaking from a conical funnel at the rate of $5cm^3/\sec$. If the radius of the base of the funnel is 5cm and the altitude is 10cm, find the rate at which the water level is dropping when it is 2.5cm from the top.

OR

Find the equation of the tangent to the curve $x^2 + 3y = 3$ which is parallel to the line $y - 4x + 5 = 0$.

16. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always a strictly increasing function in $\left(0, \dfrac{\pi}{4}\right)$.

17. If the function $f(x) = \begin{cases} 3ax + b & if\ x > 1 \\ 11 & if\ x = 1 \\ 5ax - 2b & if\ x < 1 \end{cases}$ is continuous at $x = 1$, find the values of a and b.

OR

For what value of k, the function $f(x) = \begin{cases} \dfrac{\sqrt{5x+2} - \sqrt{4x+4}}{x - 2}, & if\ x \neq 2 \\ k & if\ x = 2 \end{cases}$ is continuous at $x = 2$?

18. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$.

19. Evaluate: $\int \sin^3(2x)dx$

20. Two cards drawn simultaneously (or without replacement) from a well shuffled pack of 52 cards. Find the mean of the number of kings.

21. Solve the differential equation $\left(\tan^{-1} y - x\right)dy = \left(1+y^2\right)dx$

OR

Solve: $\left(x\log x\right)\left(\dfrac{dy}{dx}\right)+y=\dfrac{2\log x}{x}$

22. If a pair coin is tossed 10 times, find the probability of (i) exactly six heads (ii) at least six heads.

23. Position vectors of ΔABC are $\vec{a}=2\hat{i}+\hat{j}+4\hat{k}, \vec{b}=3\hat{i}-\hat{j}+5\hat{k}, \vec{c}=5\hat{i}-2\hat{j}+3\hat{k}$. Find the area of ΔABC.

SECTION.D

Questions 24 to 29 carry 6 marks each.

24. Find the area of the region enclosed between the two circles $x^2+y^2=4$ and $\left(x-2\right)^2+y^2=4$.

25. Solve the system of equations by matrix method: $x+2y+z=7; x+3z=11; 2x-3y=1$

OR

Find A^{-1} if $A=\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the equations:

$x+2y-3z=-4, 2x+3y+2z=2; 3x-3y-4z=11$.

26. Find the vector equation of the line passing through the point (1, 2,-4) and perpendicular to the two lines: $\dfrac{x-8}{3}=\dfrac{y+19}{-16}=\dfrac{z-10}{7}$ and $\dfrac{x-15}{3}=\dfrac{y-29}{8}=\dfrac{z-5}{-5}$.

27. If $f: R-\{3/5\} \to R-\{3/5\}$ be a function defined by $f(x)=\dfrac{3x+2}{5x-3}, x\in R-\{3/5\}$. Show that $f^{-1}(x)=f(x), x\in R-\{3/5\}$

OR

Let $f: N \cup \{0\} \rightarrow N \cup \{0\}$ be defined by $f(n) = \begin{cases} n+1, if\ n\ is\ even \\ n-1, if\ n\ is\ odd \end{cases}$. Show that f is invertible and $f = f^{-1}$

28. Evaluate: $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

OR

Evaluate: $\int_0^{\pi} \frac{x\, dx}{1+\sin x}$

29. A producer has 30 units of labour and 17 units of capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly 3 units of labour and 1 unit of capital are required to produce one unit of Y. If X and Y are priced at rupees 100 and rupees 120 per unit respectively, how should the producer use his resources to maximize the total revenue? Solve the problem graphically. What quality of the producer is depicted in his activity?

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