MARKING SCHEME – SET –III

SECTION – A

1. a^5b^5. (1 Mark)
2. Ans : $\frac{5}{2}$ (1 Mark)
3. No (1mark)
4. Ans: $\sqrt{a^2 + b^2}$ (1 Mark)
5. Ans: 7.2 cm (1 Mark)
6. Ans : $A + 25 + B = 90$

   $A + B = 65$ (1 Mark)

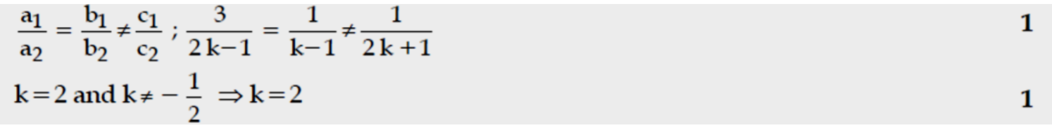
SECTION – B

7. Ans:

   $11(7 \times 13 + 1)$ (1mark)

   Yes, It is expressed as a product of two factors (1mark)

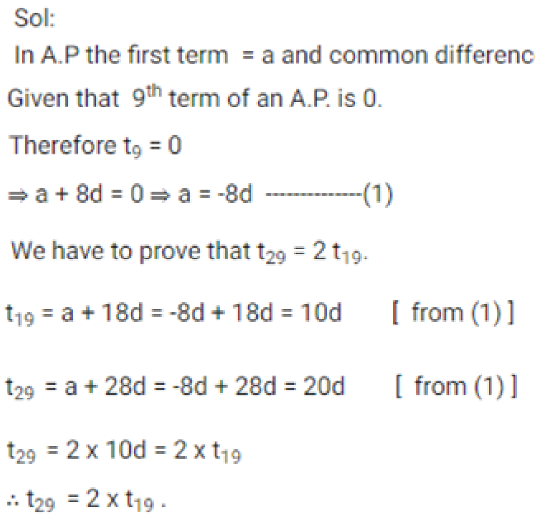
8. Ans:



$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$; $\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$ 1

$k=2$ and $k \neq -\frac{1}{2}$ $\Rightarrow k=2$ 1

9.



Sol:

In A.P the first term = a and common difference = d.

Given that 9th term of an A.P. is 0.

Therefore $t_9 = 0$

$\Rightarrow a + 8d = 0 \Rightarrow a = -8d$ --------------(1)

1/2 mark

We have to prove that $t_{29} = 2\,t_{19}$.

$t_{19} = a + 18d = -8d + 18d = 10d$ [from (1)] 1/2 mark

$t_{29} = a + 28d = -8d + 28d = 20d$ [from (1)] 1/2 mark

$t_{29} = 2 \times 10d = 2 \times t_{19}$

$\therefore t_{29} = 2 \times t_{19}$.

1/2 mark

10. Ans:

    Let the ratio be k:1

    $3\left(\frac{4k+2}{k+1}\right) + 2\left(\frac{7k+1}{k+1}\right) = 10$ (1mark)

    $26k - 10k = 10 - 8$ ($\frac{1}{2}$ mark)

Then the ratio is 1:8 ($\frac{1}{2}$ mark)

11. Ans: The possible outcomes are$\{\mathrm{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}\}$ $\frac{1}{2}$ *mark*

(i) $\frac{1}{8}$ $\frac{1}{2}$ *mark*

(ii) $\frac{3}{8}$ $\frac{1}{2}$ *mark*

(iii) $\frac{4}{8} = \frac{1}{2}$ $\frac{1}{2}$ *mark*

12. Ans: Let x be number of green balls

$p(x) = \frac{2}{3}$ $\frac{1}{2}$ *mark*

$\frac{x}{24} = \frac{2}{3}, x = 16$ 1 mark

So, number of blue balls = $24 - 16 = 8.$ $\frac{1}{2}$ *mark*

SECTION –C

13.

Let us suppose that $\sqrt{6} + \sqrt{2}$ is rational

Then $\sqrt{6} + \sqrt{2} = \frac{a}{b}$ where a and b are integers ½

$8 + 2\sqrt{12} = \frac{a^2}{b^2}$

$8 + 4\sqrt{3} = \frac{a^2}{b^2}$ ½

$\sqrt{3} = \frac{\frac{a^2}{b^2} - 8}{4}$ ½

½

$\therefore$ a, b and 6 are integers then $\frac{\frac{a^2}{b^2} - 8}{4}$ is rational

$\Rightarrow \sqrt{3} =$ rational

But we know that $\sqrt{3}$ is irrational ½

$\Rightarrow$ irrational $=$ rational which is a contradiction ½

Hence $\sqrt{6} + \sqrt{2}$ is irrational.

14.

$x = 2 \pm \sqrt{3}$ are the zeroes of $p(x)$, so

$x - \left(2 \pm \sqrt{3}\right)$ are the factors of $p(x)$.

Now, $\left\{x - \left(2 + \sqrt{3}\right)\right\}\left\{x - \left(2 - \sqrt{3}\right)\right\}$

$= \left\{\left(x-2\right) - \sqrt{3}\right\}\left\{\left(x-2\right) + \sqrt{3}\right\}$

$= \left(x-2\right)^2 - \left(\sqrt{3}\right)^2$

$= x^2 - 4x + 1$ 1 mark

Dividing $p(x)$ by $x^2 - 4x + 1$

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

1 mark

$x^2 - 2x - 35 = (x-7)(x+5)$

Zeroes are 7,-5, $2 \pm \sqrt{3}$ (1 mark)

15.

Ans: The line y = 4 $(\frac{1}{2}\ marks)$

The line $2x + y = 6$, points (0,6) and (3,0) $(1\ marks)$

The vertices are (0,0),(3,0),(1,4) and (0,4) $(\frac{1}{2}\ marks)$

Area $= \frac{1}{2} \times 4(1+7) = 16\ square\ units$ (1 mark)

16. Show that the points $(-4,-1), (-2,-4), (4,0) and\ (2,3)$ are the vertices of a rectangle.

AB = $\sqrt{13}$ units $(\frac{1}{2}\ marks)$

BC = $\sqrt{52}$ units $(\frac{1}{2}\ marks)$

CD = $\sqrt{13}$ units $(\frac{1}{2}\ marks)$

DA = $\sqrt{52}$ units $(\frac{1}{2}\ marks)$

AC = $\sqrt{65}$ units $(\frac{1}{2}\ marks)$

BD = $\sqrt{65}$ units ($\frac{1}{2}$ $marks$)

OR

Ans:

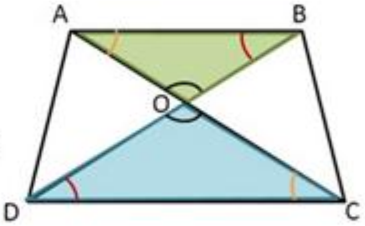
$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$ 1 mark

$x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$ 1 mark

$x - y - 2 = 0$ 1 mark

17.

Ans:



Given: ABCD is a trapezium with

AB II CD

and diagonals AB & CD intersecting at O

To prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof: In $\Delta\, OAB$ and $\Delta\, OCD$

$\angle AOB = \angle DOC$ *(Vertically opposite angles)* 1 mark

$\angle ABO = \angle CDO$ *(since AB II CD with BD as traversal, alternate angle are equal*

$\Delta\, OAB \sim \Delta\, OCD$ *(AA Similarity)* 1 mark

We know that If 2 triangles are similar ,

their corresponding sides are in proportion

Hence

$\frac{OA}{OC} = \frac{OB}{OD}$ 1 mark

Hence proved

OR

In ΔADB

$AB^2 = BD^2 + AD^2$ (Pythagoras theorem)

$\Rightarrow AD^2 = AB^2 - BD^2$ – (1) **1**

Also in ΔADC

$AC^2 = CD^2 + AD^2$ (Pythagoras theorem) **1**

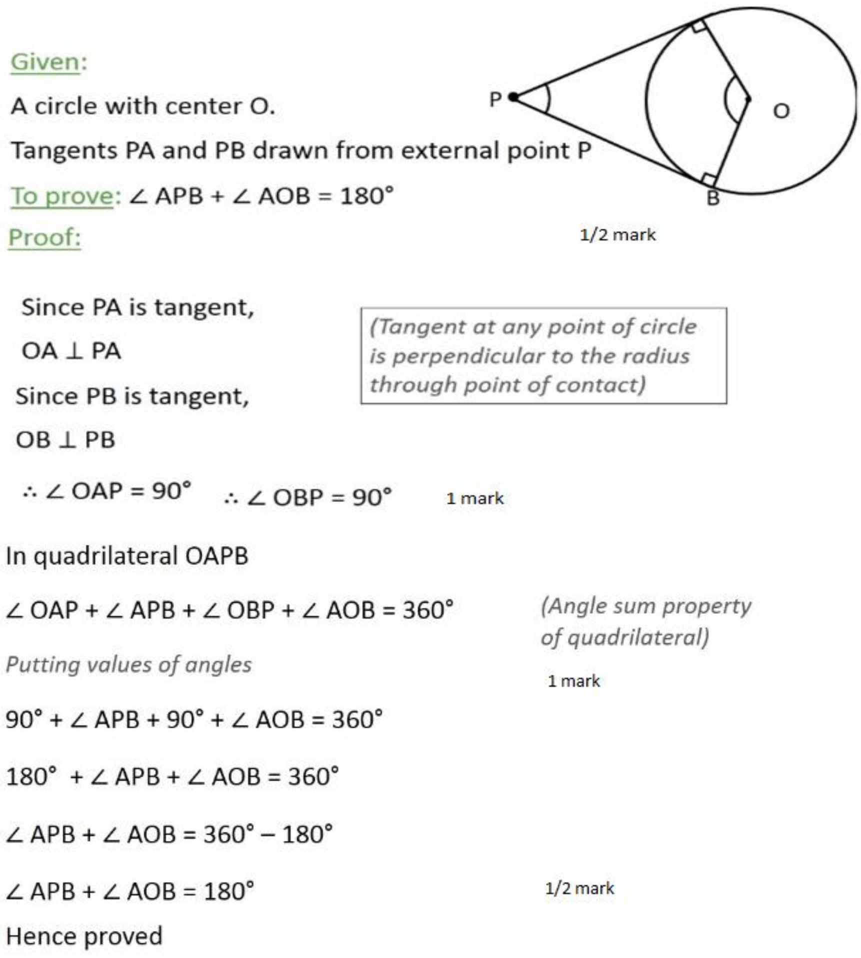
$\Rightarrow AD^2 = AC^2 - CD^2$ – (2)

From (1) and (2)

$AB^2 - BD^2 = AC^2 - CD^2$

$\Rightarrow AB^2 + CD^2 = BD^2 + AC^2$ **1**

18. Ans:



Given:

A circle with center O.

Tangents PA and PB drawn from external point P

To prove: $\angle APB + \angle AOB = 180^\circ$

Proof:

1/2 mark

Since PA is tangent,

$OA \perp PA$

Since PB is tangent,

$OB \perp PB$

(Tangent at any point of circle is perpendicular to the radius through point of contact)

$\therefore \angle OAP = 90^\circ$ $\therefore \angle OBP = 90^\circ$ 1 mark

In quadrilateral OAPB

$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360^\circ$ *(Angle sum property of quadrilateral)*

Putting values of angles

1 mark

$90^\circ + \angle APB + 90^\circ + \angle AOB = 360^\circ$

$180^\circ + \angle APB + \angle AOB = 360^\circ$

$\angle APB + \angle AOB = 360^\circ - 180^\circ$

$\angle APB + \angle AOB = 180^\circ$ 1/2 mark

Hence proved

19.

$m^2 - n^2 = (m+n)\ (m-n)$

$= [(\tan\theta + \sin\theta) + \tan\theta - \sin\theta]$

$[\tan\theta + \sin\theta - \tan\theta + \sin\theta]$

$= 2\tan\theta\ 2\sin\theta$

$= 4\tan\theta\ \sin\theta$

$= 4\sqrt{\tan^2\theta\ \sin^2\theta}$ 1

$= 4\sqrt{\left(\sec^2\theta - 1\right)\ \sin^2\theta}$

$= 4\sqrt{\sec^2\theta\ \sin^2\theta\ -\ \sin^2\theta}$ 1

$= 4\sqrt{\tan^2\theta\ -\ \sin^2\theta}$

$= 4\sqrt{mn}$

$(m^2 - n^2)^2 = 16\ mn$ 1

OR

Ans:

$\frac{2\cos(90^\circ - 68^\circ)}{\cos22^\circ} - \frac{2\tan(90^\circ - 15)}{5\tan(90^\circ - 15)}$ 1 mark

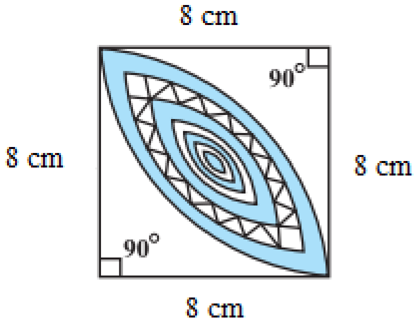
$- \frac{3\times 1\cot(90^\circ - 20^\circ)\cot(90^\circ - 40^\circ)\tan50^\circ\tan70^\circ}{5\left\{\cos^2(90^\circ - 70^\circ) + \sin^2 20^\circ\right\}}$

$\Rightarrow \frac{2\cos22}{\cos22} - \frac{2}{5} - \frac{3\cot70^\circ\cot50^\circ\tan50^\circ\tan70^\circ}{5(\cos^2 20^\circ + \sin^2 20^\circ)}$ 1/2 mark

$\Rightarrow 2 - \frac{2}{5} - \frac{3}{5}$ 1/2 mark

$\Rightarrow \frac{10-2-3}{5} = \frac{5}{5} = 1$ 1 mark

20.



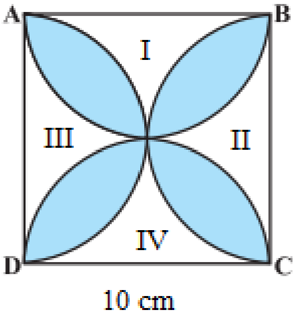
Ans:

$2(\ sector\ area - triangle\ area)$ 1 mark

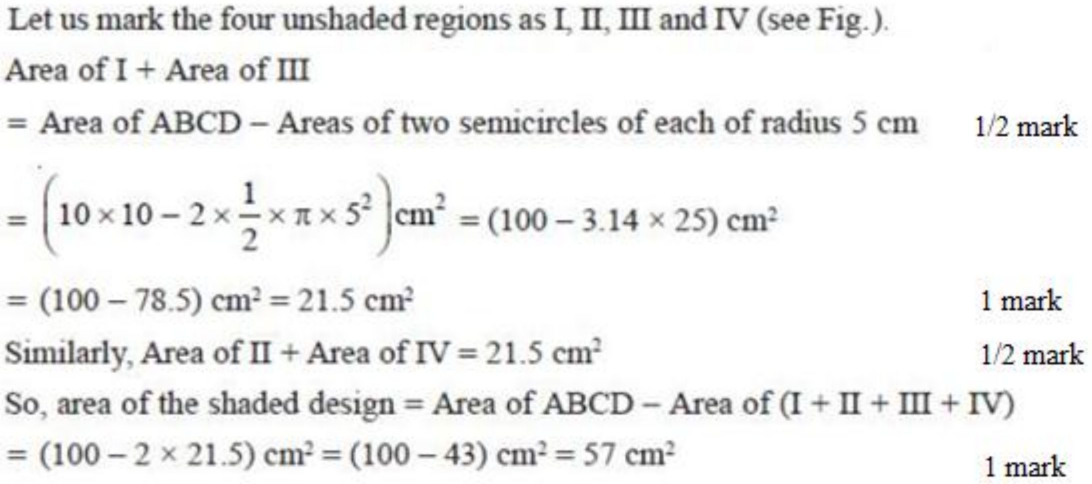
$2(\frac{1}{4}\times\frac{22}{7}\times 8^2 - \frac{1}{2}\times 8\times 8)$ 1 mark

$\frac{256}{7}$ 1 mark

OR



Ans:



Let us mark the four unshaded regions as I, II, III and IV (see Fig.).

Area of I + Area of III

= Area of ABCD – Areas of two semicircles of each of radius 5 cm 1/2 mark

$= \left(10 \times 10 - 2 \times \frac{1}{2} \times \pi \times 5^2\right) \text{cm}^2 = (100 - 3.14 \times 25) \text{ cm}^2$

$= (100 - 78.5) \text{ cm}^2 = 21.5 \text{ cm}^2$ 1 mark

Similarly, Area of II + Area of IV = 21.5 cm² 1/2 mark

So, area of the shaded design = Area of ABCD – Area of (I + II + III + IV)

$= (100 - 2 \times 21.5) \text{ cm}^2 = (100 - 43) \text{ cm}^2 = 57 \text{ cm}^2$ 1 mark

21. Ans:

Volume of ice cream cone = $\frac{volume\ of\ cylinder}{Number\ of\ ice\ cream\ cones}$ $(\frac{1}{2}\ marks)$

$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{\pi R^2 H}{10}$ $\frac{1}{2}\ marks)$

$\frac{1}{3}\pi r^3(4+2) = \frac{\pi R^2 H}{10}$

$r^3 = 27$

$r = 3\ cm$ $(1\frac{1}{2}\ marks)$

$Diameter = 6\ cm$ $(\frac{1}{2}\ marks)$

22. Ans :

Correct table (1 mark)

Correct graph (1 mark)

Median = 138 (1 mark)\

SECTION –D

23. Ans:

The time taken to fill a cistern by smaller pipe = x minutes

Work done by smaller pipe in 1 minute = $\frac{1}{x}$

The time taken to fill a cistern by larger pipe = x-3 minutes

Work done by larger pipe in 1 minute = $\frac{1}{x-3}$

The total time taken by both the pipes together to fill the cistern = $3\frac{1}{13} = \frac{40}{13}\ minutes$

Work done by both in 1 minute = $\frac{13}{40}$

According to the problem,

$\frac{1}{x} + \frac{1}{x-3} = \frac{13}{40}$ 1 mark

$13x^2 - 119x + 120 = 0$ $\frac{1}{2}\ mark$

$$b^2 - 4ac = 7921$$

$$\sqrt{7921} = 89$$

$x = \frac{-(-119)\pm 89}{26} = 8$ or $\frac{30}{26}$ 1 mark

Rejecting $x = \frac{30}{26}$ (since less than 3) $\frac{1}{2}\ mark$

Hence, the larger pipe takes $8 - 3$ = 5 minutes and smaller pipe takes $x = 8\ minuts$ (1 mark)

OR

Let the speed of the stream be x km/hr

$\frac{12}{11-x} + \frac{12}{11+x} = \frac{11}{4}$ 1 mark

$x^2 - 121 + 96 = 0$ $1\frac{1}{2}\ marks$

$x \neq -5\ , x = 5$ 1 mark

$speed\ of\ stream = 5\ km/h$ $\frac{1}{2}\ mark$

24. Ans: Number of trees planted by class I = 3

Number of trees planted by class II = 3 $\times 2 = 6$

Number of trees planted by class III = 3 $\times 3 = 9$

…

Number of trees planted by class XII = 12 × 3 = 36 (2 marks)

The number of trees form an A.P

i.e 3,6,9,12,.. , 36 , $n = 12$

$S_n = \frac{12}{2}$ ($3 + 36$) = 234 (1mark)

Hence the total number of trees that will be planted by the students is 234.

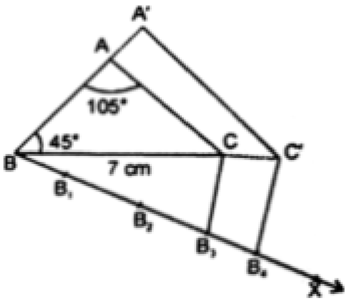
(b) Yes. Planting more trees helps in reducing pollution and thus, make environment clean and green. (1 mark)

25. Ans:

Given, To prove, Diagram, Construction 1 mark

Correct Proof 3 marks

26. Ans:



Construction of triangle ABC	1 mark
Construction of similar triangle	3 marks

27.

$$\text{LHS } \frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}+\frac{\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}$$

$$\frac{\sin A\times\sin A}{\cos A\,(\sin A-\cos A)}+\frac{\cos A\times\cos A}{\sin A\,(\cos A-\sin A)}$$ 1 mark

$$\frac{\sin^2 A}{\cos A\,(\sin A-\cos A)}-\frac{\cos^2 A}{\sin A\,(\sin A-\cos A)}$$

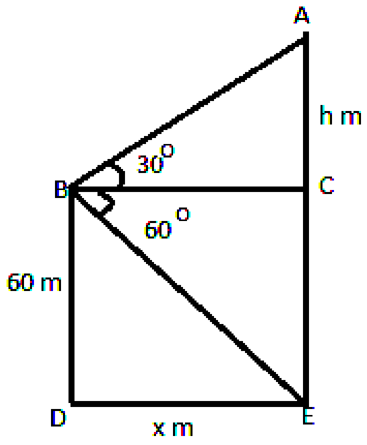
$$\frac{\sin^3 A-\cos^3 A}{\sin A\ \cos A\,(\sin A-\cos A)}$$ 1 mark

$$\frac{\left(\cancel{\sin A-\cos A}\right)\left(\sin^2 A+\cos^2 A+\cos A\sin A\right)}{\sin A\ \cos A\left(\cancel{\sin A-\cos A}\right)}$$ 1 mark

$$\frac{1+\cos A\sin A}{\sin A\cos A}=\frac{1}{\sin A\cos A}+1$$ 1 mark

$$=\sec A\,\mathrm{cosec}A+1-\text{RHS}$$

28.



Let BD = 60 m be the height of the building.

Let A nd E the top and bottom position of the light house.

Let h be the difference of heights of the building and the light house.

Let x be the distance between light house and the building.

Consider the triangle ABC

In $\Delta ABC, \quad BC = \sqrt{3}h$ (1 Mark)

In $\Delta BCRE, \quad BC = \frac{60}{\sqrt{3}}$ (1 Mark)

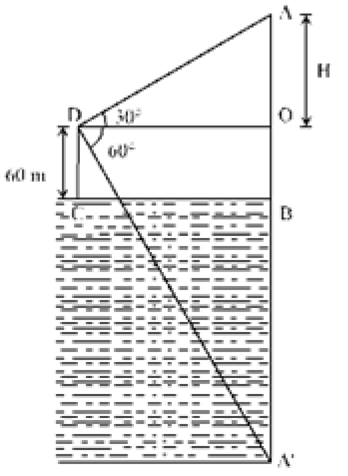
$h = 20\ m$ (1 Mark)

BC =20 $\sqrt{3}$ m $(\frac{1}{2}\ Marks)$

Difference between the height = 20 m

distance between them = $20\sqrt{3}$ $\frac{1}{2}\ marks$

OR



Let AO = H

CD = OB = 60 m

A' B = AB = 60 + H

In Δ AOD,

$\tan 30° = \frac{AO}{OD} = \frac{H}{OD}$

$H = \frac{OD}{\sqrt{3}}$

$OD = \sqrt{3}\,H$ 1 mark

Now, In Δ A'OD,

$\tan 60° = \frac{OA'}{OD} = \frac{OB+BA'}{OD}$

$\sqrt{3} = \frac{60+60+H}{\sqrt{3}H} = \frac{120+H}{\sqrt{3}H}$ 1 mark

$\Rightarrow 120 + H = 3H$

$\Rightarrow 2H = 120$

$\Rightarrow H = 60$ m 1 mark

Thus, height of the cloud from the surface of the lake = AB + A'B = 60 + 60 = 120 m. 1 mark

29. Ans:

$l = 31.62\ cm$	1 mark
S.A = 3295.6 cm^2	1 mark
Volume = 22000 cm^3	1 mark
Cost of milk = Rs: 550	1 mark

30.

Marks	Frequency	x_i	$u_i = \frac{x_i - 67.5}{5}$	$f_i u_i$
50-55	5	52.5	-3	-15
55-60	8	57.5	-2	-16
60-65	15	62.5	-1	-15
65-70	11	67.5	0	0
70-75	7	72.5	1	7
75-80	4	77.5	2	8
	50			-31

(2 Marks)

$\bar{u} = \frac{-31}{50}$

$\bar{x} = a + h\bar{u}$ = 64 .4 (2 Marks)

OR

x_i	Number of Workers	$u_i = \frac{x_i - 45}{10}$	$f_i u_i$
5	5	-4	-20
15	7	-3	-21
25	$\boldsymbol{f_1}$	-2	-2f_1
35	3	-1	-3
45	$\boldsymbol{f_2}$	**0**	**0**
55	9	1	9
65	6	2	12
$Total$	30 $+f_1 + f_2$		-23-2f_1

(1 Mark)

$f_1 + f_2 = 70$ (1 mark)

$f_1 = 20$ (1 mark)

f_2 = 50 (1 mark)