# INDIAN SCHOOL DARSAIT

**Class XII** ***Mathematics Worksheet***

## *Worksheet # 3 Binary Operations*

## *(Chapter – 1: Relations & Functions)*

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| | ***CLASS WORK*** |
| 1. | State which of the following operations are binary?<br>i) $a*b$ = a + ab, a, b $\in$ Q<br>ii) $a*b$ = a + $4b^2$, a,b $\in$ R<br>iii) $a*b$ = $a^3 + b^3$, a, b $\in$ N<br>iv) $a*b$ = a – b + ab, a, b $\in$ Z |
| 2. | Check whether the following operations defined on the given set are commutative and associative:<br>i) $a*b = \frac{a}{b+1}$, a,b $\in R - \{-1\}$ iv) $a*b$ = 1, a,b $\in$ N<br>ii) $a*b = \frac{a+b}{2}$, a, b $\in$ N<br>iii) $a*b$ = a – b + ab, a, b $\in$ Z |
| 3. | On Q, the set of rational numbers, an operation $*$ is defined by $a*b = \frac{ab}{5}$ for all a, b $\in$Q. Show that $*$ is i) a binary operation ii) commutative and associative. Find the identity element for $*$ in Q. Also prove that every non – zero element of Q is invertible |
| 4. | Let $*$ be an operation on the set $Q - \{1\}$, defined by $a*b = a+b-ab$ for all a, b $\in Q - \{1\}$. Check whether $*$ is commutative and associative. Find the identity element for with respect to $*$. Also prove that every element of $Q - \{1\}$ is invertible? |
| 5. | Let A = $N\cup\{0\} \times N\cup\{0\}$ and $*$ be a binary operation on A defined by (a,b) $*$ (c,d) = (a+c, b+d) for all (a,b) , (c,d) $\in$ A. Show that $*$ is commutative and associative. Also find the identity element for $*$ in A. |
| 6. | Let A = N × N and $*$ be a binary operation on A defined by (a, b) $*$ (c, d) = (ad + bc, bd) for all (a, b), (c, d) $\in$ A. Show that i) * is commutative<br>ii) * is associative iii) has no identity element |
| 7. | Let * be a binary operation on N by a * b = LCM of a and b for all a, b$\in$N.<br>i) Find 5*7, 20*16 ii) Is * commutative and associative?<br>iii) Find the identity element in N w.r.to *<br>iv) Which are the invertible elements of N? |
| 8. | Let X be a non – empty set and * be a binary operation defined on P(X), the power set of X, defined by A*B = A$\cup$B, for all A, B $\in$P(X).<br>i) Prove that * is commutative and associative<br>ii) Find the identity element w.r.t * iii) Show that $\phi$ is the invertible element<br>If O is another operation defined on P(X) by AOB = A $\cap$ B for all A, B $\in$P(X).<br>Show that * is distributive over O. |
| 9. | Define a binary operation * on the set {0,1, 2, 3, 4, 5} as $a*b = \begin{cases} a+b, & if\ a+b<6 \\ a+b-6, & if\ a+b \geq 6 \end{cases}$.<br>Show that i) 0 is the identity for this operation<br>ii) each element of a is invertible with 6 – a is the inverse. |

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| 10. | Consider the binary operations *, o : R × R→R defined as a*b = \|a – b\| and aob = a for all a, b ∈R. Show that i) * is commutative but not associative<br>ii) o is associative but not commutative<br>iii) * is distributive over o |
| 11. | Consider the binary operation ∗ on the set {1, 2, 3, 4, 5} defined by a ∗ b = HCF of a and b.<br>i) Write the operation table.<br>ii) Is ∗ commutative?<br>iii) Also compute (2∗3)∗5 & (2∗3)∗(4∗5) |
| 12. | A binary operation * is defined on the set by $a*b=\begin{cases} a, & if \quad b=0 \\ \|a\|+b, & if \quad b\neq 0\end{cases}$. If at least one of a and b is 0, then prove that $a*b=b*a$. Check whether ∗ is commutative. Also find the identity element w.r to ∗ if it exists. |
| 13. | On the set M = A(x) = $\left\{\begin{bmatrix} x & x \\ x & x\end{bmatrix} : x\in R\right\}$ of 2×2 matrices, find the identity element for the binary operation "Multiplication of matrices". Also find inverse of each element of M. |
| | ***HOME WORK*** |
| 14. | Check whether the following operations defined on the given set are commutative and associative: -<br>i) $a*b$ = $2^{ab}$, a,b ∈ Q<br>ii) $a*b$ = $a^3 + b^3$, a, b ∈ N<br>iii) $a*b$ = ab + 1, ab ∈ Q |
| 15. | Let ∗ be an operation on$Q_0$, the set of non – zero rational numbers, defined by $a*b=\frac{ab}{4}$ for all a, b ∈$Q_o$. Show that ∗ is i) a binary operation ii) commutative and associative. Find the identity element for ∗ in Q. What is the inverse of each element of $Q_o$? |
| 16. | On the set R – {-1}, an operation ∗ is defined by $a*b=a+b+ab$for all a, b ∈R – {-1}. Prove that ∗ is i) a binary operation ii) commutative as well as associative. Find the identity element for with respect to ∗. Also prove that every element of R – {-1} is invertible? |
| 17. | Let ∗ be an operation on $R_0$, the set of non – zero real numbers, defined by $a*b=\frac{ab}{3}$ for all a, b ∈$Q_o$. Find the value of x, given that 2 ∗ ( x ∗ 5) = 10 |
| 18. | Let $R_0$ be the set of all non – zero real numbers and A = $R_0$× $R_0$. Let ∗ be a binary operation on A defined by (a,b) ∗ (c,d) = (ac, bd) for all (a,b) , (c,d) ∈ A.<br>i) Show that ∗ is commutative and associative<br>ii) Find the identity element for ∗ in A iii) Find the invertible elements in A |
| 19. | Let A = Q × Q and ∗ be an operation defined on A by (a,b) ∗ (c,d) = (ac, b+ad) for all (a,b) , (c,d) ∈ A. Determine whether ∗ is binary. If so find the identity element in A. Also find the invertible elements in A. |

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| 20. | Let X be a non – empty set and * be a binary operation defined on P(X), the power set of X, defined by A*B = A$\cap$B, for all A, B $\in$P(X).<br>i) Prove that * is commutative and associative ii) Find the identity element w.r.t *<br>iii) Show that X is the invertible element. If O is another operation defined on P(X) by AOB = A $\cup$ B for all A,B $\in$P(X). Show that * is distributive over O. |
| 21. | Let X be a non – empty set and * be a binary operation defined on P(X), the power set of X, defined by A*B = (A – B)$\cup$(B – A), for all A, B $\in$P(X).<br>Prove that i) $\phi$ is the identity element w.r.t * in P(X)<br>ii) A is invertible for all A $\in$P(X) and $A^{-1}$ = A. |
| 22. | Define a binary operation * on the set {0,1, 2, 3, 4, 5,6} as $a*b=\begin{cases} a+b, & if\ \ a+b<7 \\ a+b-7, & if\ \ a+b\geq 7 \end{cases}$.<br>Show that i) Write the operation table<br>ii) 0 is the identity for this operation<br>iii) each element of a is invertible with 6 – a is the inverse. |
| 23. | Define a binary operation * on the set A = {0,1, 2, 3, 4, 5} as a*b = ab(mod5).<br>Show that i) 1 is the identity with respect to *<br>ii) All elements of A are invertible with $2^{-1}$ = 3 and $4^{-1}$ = 4 |
| 24. | Let $*$ be a binary operation defined on the set Z of integers by $a*b=a+b-5$ for all a, b $\in$ Z. Show that $*$ is commutative and associative. Also find the identity element if it exists. |
| 25. | Give an example of a binary operation which is<br>i) commutative as well as associative<br>ii) commutative but not associative<br>iii) associative but not commutative |
| 26. | Let $*$ be an operation defined on the set Z of integers by $a*b=a+b+2$ for all a,b $\in$ Z. i) Prove that $*$ is a binary operation.<br>ii) Show that $*$ is commutative and associative.<br>iii) Find the identity element w.r.t $*$ on Z iv) Find the inverse of a $\in$ Z. |
| **SELF STUDY** | |
| 27. | Is $*$ defined on the set A = {1, 2, 3, 4, 5} by a $*$ b = LCM of a and b, a binary operation? Justify your answer. |
| 28. | A binary operation * on R – {-1} defined as $a*b=\dfrac{a}{b+1}$. Is $*$ commutative and associative? Justify your answer. |
| 29. | Consider the binary operation $*$ on the set A = {1, 2, 3, 4, 5} defined by<br>a $*$ b = Min {a, b}. Write the operation table. |
| 30. | Let $*$ be a binary operation defined on the set Q of rational numbers by $a*b=\dfrac{3ab}{5}$<br>Show that $*$ is commutative and associative. Also find the identity element if it exists. |
| 31. | On the set $Q_+$ of all positive rational numbers define the operation $*$ by<br>$a*b=\dfrac{ab}{3}$, a, b $\in$ $Q_+$.<br>i) Show that $*$ is a binary operation iii) Find the identity element w.r.t $*$<br>ii) Show that $*$ is commutative and associative iv) What is the inverse of a $\in$ $Q_+$. |

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| 32. | Consider the binary operation $*$ on the set A = {6, 7, 8, 9, 10} defined by $a * b = \text{Min}\{a, b\}$. Write the operation table. |
| 33. | If $A = R - \{0\}$ and $*$ be a binary operation defined on A by $a*b = 2ab$, $\forall a,b \in A$.<br>Then i) Show that $*$ is commutative<br>ii) Show that $*$ is associative<br>iii) Write the identity element w.r.t $*$ on A<br>iv) If the inverse exists, find the inverse of a. |